MEASURING POLYWELL™ CUSP CONFINEMENT WITH A RELATIVISTIC PARTICLE CODE

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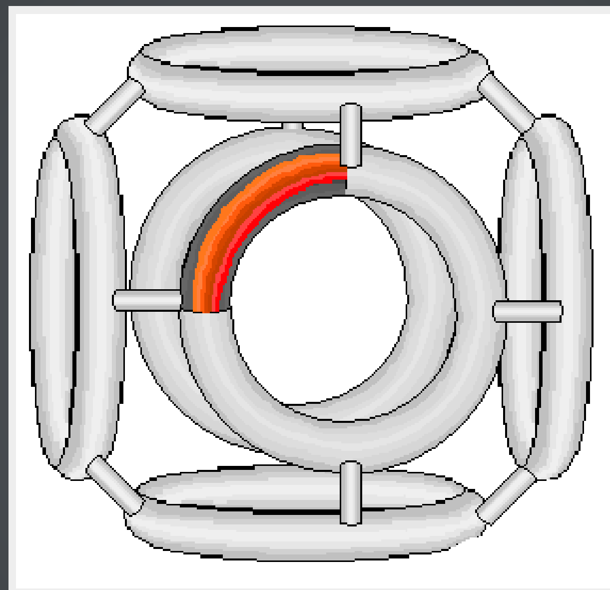


OPEN QUESTIONS

Electron confinement time
Ion confinement
Thermalisation
Geometry-dependence
Scaling with radius, energy

PHYSICAL CONDITIONS INSIDE A POLYWELL

1. Electrons: $E \approx 100\text{keV}$ near edge of device (Relativistic!)
2. Electrons: $E \ll 100\text{keV}$ near core of device
3. Ions: $E \approx 500\text{keV}$ near core of device, very low on edge of device
4. Nonequlibrium regions, Nonadiabatic regions
5. Low densities ($10^{10} cm^{-3}$)
6. Intrinsically 3D geometry

CONSEQUENCES

1. λ_D order of device away from core
2. $\lambda_D \approx 1mm$ near core of device
3. $N_D \approx 10^8 - 10^{13}$ over device
4. Departures from quasi-neutrality

APPROACHES CONSIDERED

What are our options?

FOKKER-PLANCK

PRO

- Allows insight into full distribution function

CON

- Only valid for small-angle scattering
- Neglects relativistic effects
- Bounce-averaging requires conservation of $\vec{\mu}$ (Polywell: B=0 in centre)
- 6-dimensional distribution function is infeasibly large!

VLASOV-MAXWELL

PRO

- Allows insight into full distribution function
- Self-consistent fields

CON

- Neglects short-distance behaviour
- Does not reproduce real entropy increase over time
- Only valid when $N_D \to \infty$ (Polywell: \approx 10^{10})
- 6-dimensional distribution function is infeasibly large!

PARTICLE-IN-CELL

PRO

- Allows insight into approximate distribution function

CON

- Approximates short-distance behaviour
- Tricky to deal with sharp, dynamic gradients
- Statistical noise is a concern
- Coarse-graining changes the collisional behaviour!
- Only valid in the limit $\Lambda_p^{PIC} \to \Lambda$

MHD

PRO

- Simple
- Little numerical noise

CON

- Assumes the distribution
- Approximates fields
- Requires highly collisional plasma

GIVE UP?

None of the above.

CHOSEN APPROACH: PARTICLE CODE

PRO

- Simple; 3D fundamentally no harder than 1 or 2D
- Can include full collisions and relativistic effects
- Exact fields
- No trouble with mixed-species plasmas

CON

- Extremely expensive!
- Naive $O(N_{particles}^2)$, Tree-Code $O(N_{particles})$

POINT SOURCE FIELDS

DEFINITIONS

$$\rho(x', t') = q\delta[x' - r(t')]$$

$$\vec{J}(x', t') = \rho\vec{v}(t')\delta[x' - r(t')]$$

$$\kappa = 1 - \frac{\vec{v} \cdot \hat{\vec{R}}}{c}$$

$$\vec{R}(t) = \vec{x}(t) - \vec{r}(t')$$

$$R = |\vec{R}|$$

$$t' = t - \frac{R(t)}{c}$$

POINT SOURCE FIELDS

$$\vec{E}(\vec{x},t) = \frac{q}{4\pi\epsilon_0}\left\{\left[\frac{\hat{\vec{R}}}{\kappa R^2}\right] + \frac{\partial}{c\partial t}\left[\frac{\hat{\vec{R}}}{\kappa R}\right] - \frac{\partial}{c^2\partial t}\left[\frac{\vec{v}}{\kappa R}\right]\right\}$$

$$\vec{B}(\vec{x},t) = \frac{\mu_0 q}{4\pi}\left\{\left[\frac{\vec{v}\times\hat{\vec{R}}}{\kappa R^2}\right] + \frac{\partial}{c\partial t}\left[\frac{\vec{v}\times\hat{\vec{R}}}{\kappa R}\right]\right\}$$

CIRCULAR COILS OF CURRENT I, CHARGE Q

$$B_\rho(\rho,z) = \frac{\mu_0 I z}{4\pi}\sqrt{\frac{m}{R\rho^3}}\left[\frac{2-m}{2-2m}E(m) - K(m)\right]$$

$$B_z(\rho,z) = \frac{\mu_0 I}{4\pi}\sqrt{\frac{m}{R\rho^3}}\left[\rho K(m) + \frac{Rm-\rho(2-m)}{2-2m}E(m)\right]$$

$$E_\rho(\rho,z) = \frac{Q}{2\pi^2\epsilon_0\sqrt{t^2-l}}\left\{\frac{\rho l - Rt^2}{l(t^2+l)}E(n) + \frac{R}{l}K(n)\right\}$$

$$E_z(\rho,z) = \frac{Qz\sqrt{1-n}}{2\pi^2\epsilon_0 (t^2+l)^{3/2}}E(n)$$

where

$$t^2 = \rho^2 + z^2 + R^2$$

$$l = 2R\rho$$

$$m = \frac{4R\rho}{z^2 + (R+\rho)^2}$$

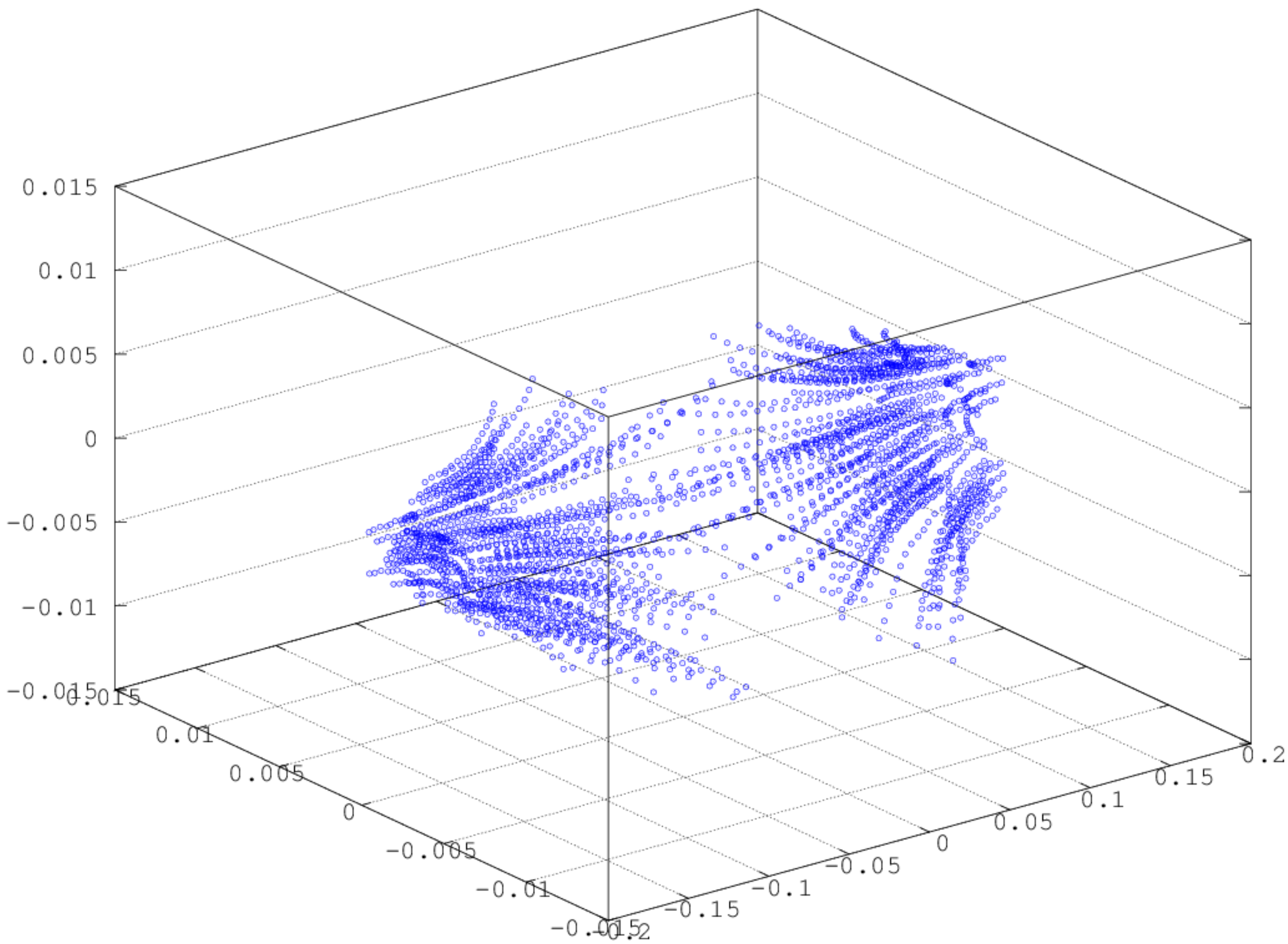
$$n = \frac{-4\rho R}{(\rho-R)^2 + z^2}$$ $

PARTICLE EVOLUTION

$$\vec{a} = \frac{\overrightarrow{F_\perp}}{m\gamma} + \frac{\overrightarrow{F_\parallel}}{m\gamma^3}$$

$$= \frac{q}{m\gamma}\left(\overrightarrow{E_\perp} + \frac{1}{\gamma^2}\overrightarrow{E_\parallel} + \vec{v} \times \vec{B}\right)$$

$$= \frac{q}{m\gamma}\left(\vec{E} + \vec{v} \times \vec{B} - \frac{v^2}{c^2}\vec{E} \cdot \overrightarrow{\hat{v}}\,\overrightarrow{\hat{v}}\right)$$



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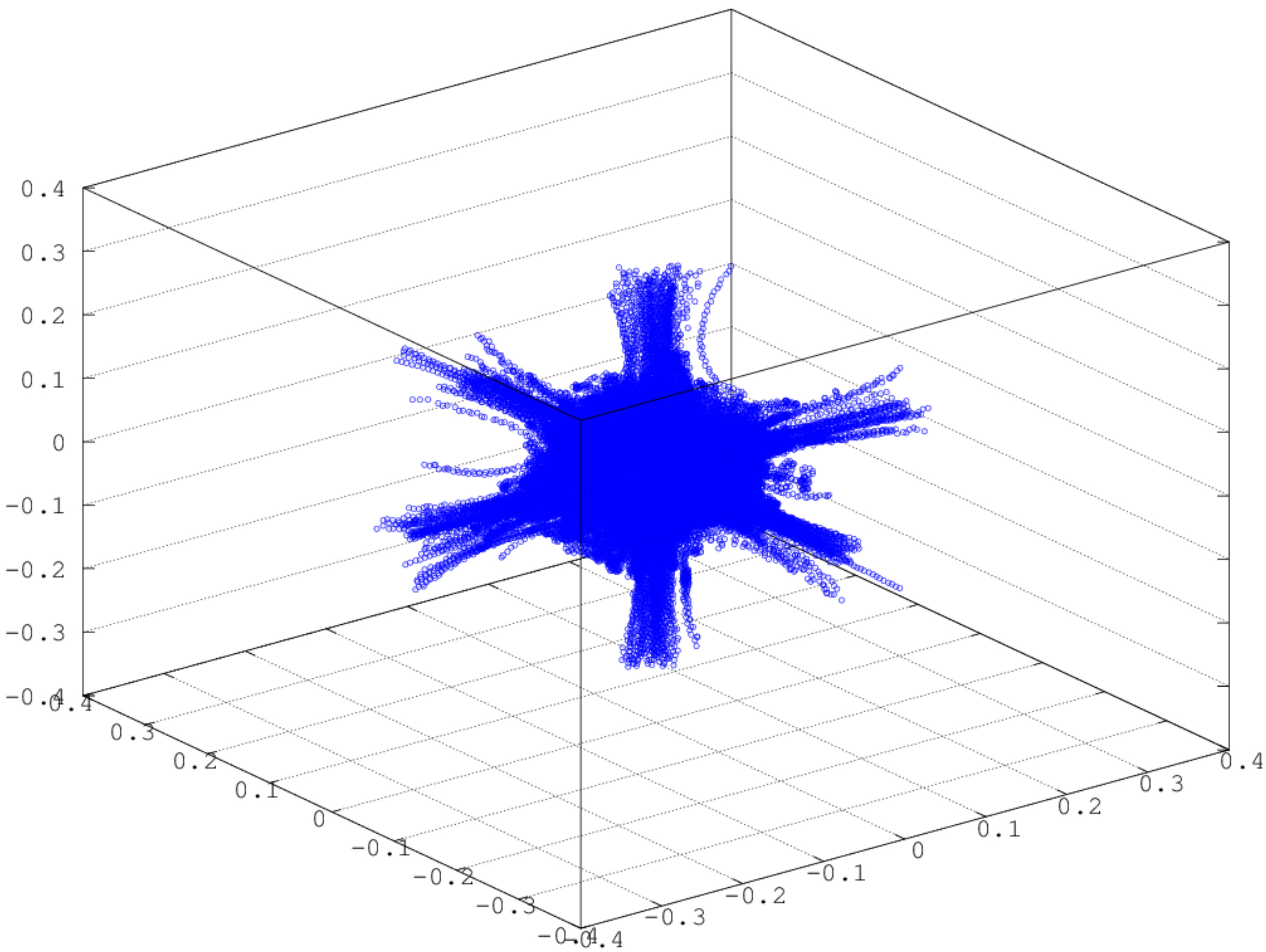
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FUTURE WORK

- Fast Multipole Method
- Testing Mirror-ratio assumption
- Estimating bounds on confinement time

LINKS

- github.com/hedj/maxrel
- Julia

THE END