Vlasov-Poisson calculations of electron confinement times in Polywell^{TM} devices using a steady-state particle-in-cell method [1] [2]

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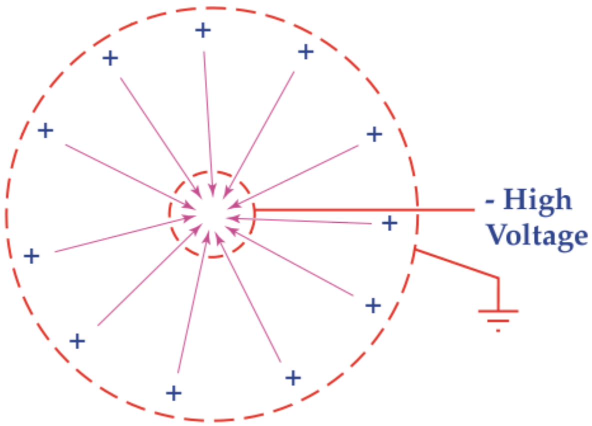
Outline

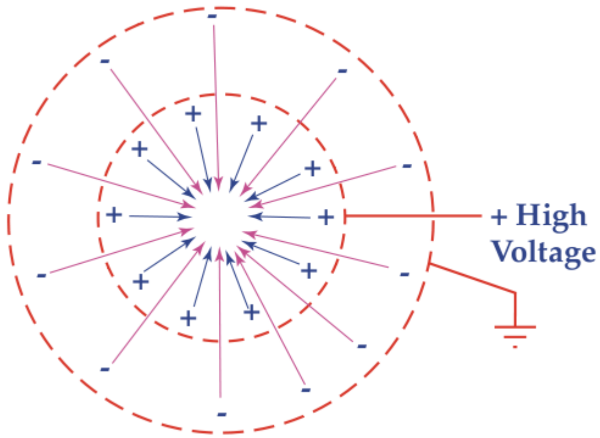
- Builds on Ph.D. thesis work of Matt Carr (University of Sydney) addressing, in part, Polywell cusp confinement without electric fields [Car13].
- A steady-state PIC strategy previously used in "gun" codes is also applicable to the Polywell concept.
- The algorithm is implemented in a small code called SSUBPIC.
- Test cases are presented (space-charge limited current, spherical galaxy).
- Polywells with **single electron species** are analyzed. Results show positively biased coils vastly improve confinement over grounded coils.

Motivation: Polywell

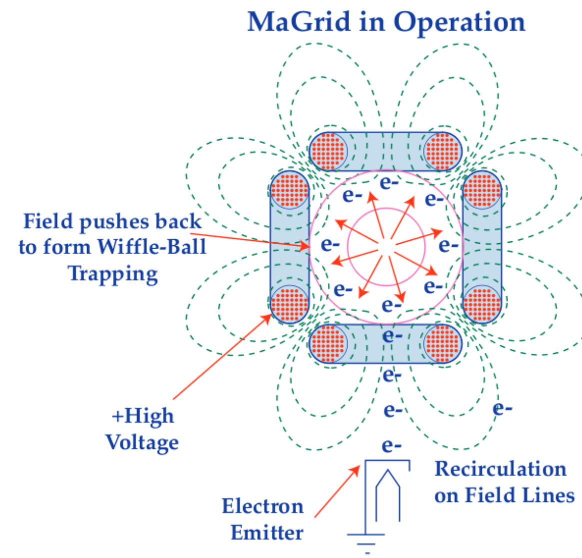
Evolution of IEC concepts leading to Polywell? [4]



Typical gridded IEC



Elmore-Tuck-Watson



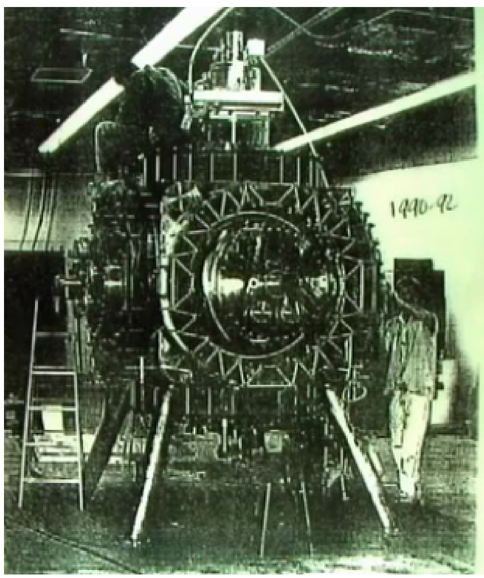
Polywell/MaGrid

Inertial electrostatic confinement (IEC) devices based on an inner high negative voltage metal grid lose many ions due to surface impact making energy break-even unlikely. Elmore, Tuck, and Watson proposed replacing inner grid with a virtual cathode of electrostatically confined electrons to avoid this [ETW59]. The Polywell confines electrons with magnetic cusps. Ions are injected inside positively biased magnetic coils and never see a solid surface until collisional up-scattering of energy.

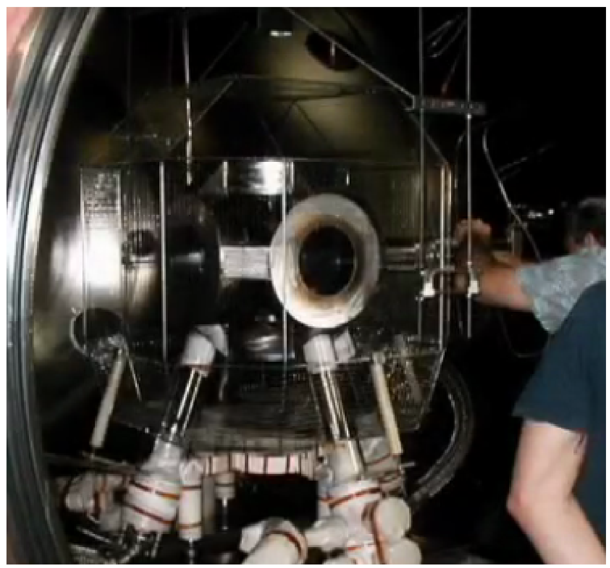
[4] Illustrations by Mark Duncan, Askmar Publishing

Motivation: Polywell

- Polywell concept proposed by Robert Bussard [Bus91, Kra92]. Early work by EMC2 company funded by DARPA and later Navy.
- Electron cusp losses are a major concern, and the subject of this work.
- Single-species electron confinement with a self-produced potential hill should closely mimic electron behavior in two-species device.
- Fusion-regime device expected to have improved electron confinement in so-called “wiffle ball” regime where core B-field is excluded.



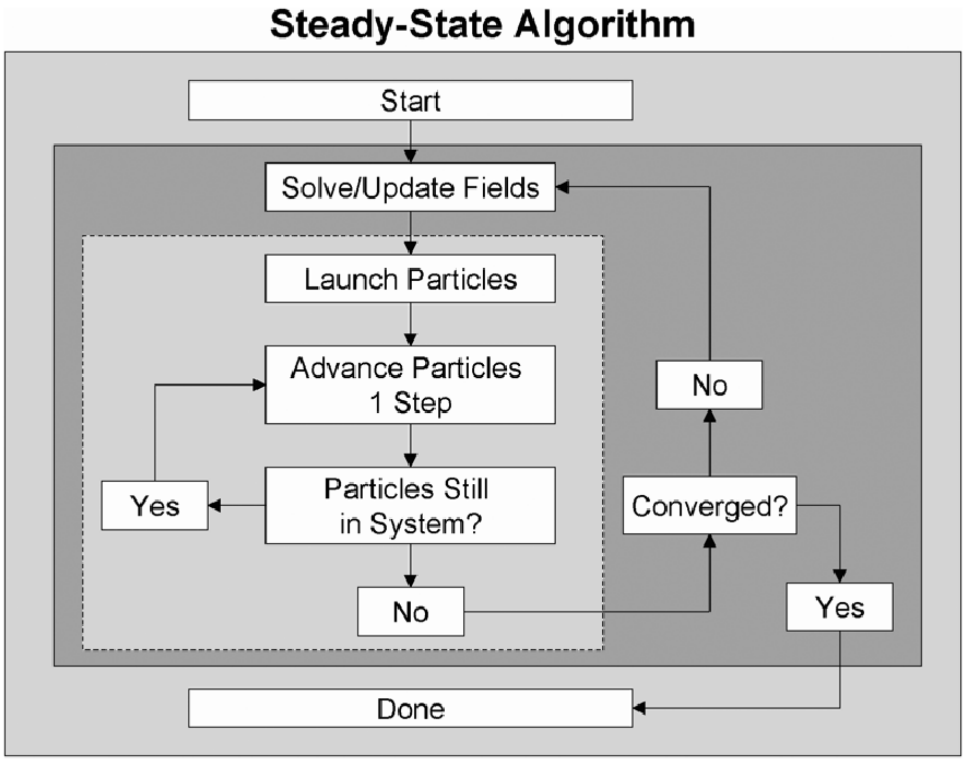
Early HEPS experiment (DARPA)


WB-4 experiment (Navy)


WB-7 experiment (Navy) features rounded coils and magnetically shielded supports.

Simple steady-state PIC algorithm

- Idea is to launch many particles in time-independent fields, weighting them to the grid at every time step.
- Solve fields using particle deposition information.
- Repeat with new set of particles in fields produced by a previous set, and so on
- Continue until particles produce the same fields (ρ, J, etc) as prior set.
- Method can be very fast compared to standard PIC because field solve is called much less often. The major disadvantage is no transient information recovered.



Scheme used by SSUBPIC and prior codes like MICHELLE. This image is taken from a Petillo et al [PND+05].

- This algorithm is used in prior codes (like MICHELLE, egun) and is implemented in our own code SSUBPIC (steady-state unstructured boundary particle-in-cell).

SSUBPIC code implementation

Straight-wire approximation: external B-field from many wire segments. Field at $\vec{x}$ due to wire from $\vec{x}_1$ to $\vec{x}_2$ is just

$$\vec{B}_{wire}(\vec{x}) = \frac{\mu_0 I \hat{\phi}(\vec{x})}{4\pi s(\vec{x})} \left(\sin\theta_2(\vec{x}) - \sin\theta_1(\vec{x})\right)$$

$$s(\vec{x}) = \frac{\|(\vec{x}_1 - \vec{x}) \times (\vec{x}_2 - \vec{x})\|}{\|\vec{x}_2 - \vec{x}_1\|}$$

$$\sin\theta_1(\vec{x}) = \frac{(\vec{x} - \vec{x}_1) \cdot (\vec{x}_2 - \vec{x}_1)}{\|\vec{x}_2 - \vec{x}_1\| \|\vec{x}_1 - \vec{x}\|}$$

$$\sin\theta_2(\vec{x}) = \frac{(\vec{x}_2 - \vec{x}) \cdot (\vec{x}_2 - \vec{x}_1)}{\|\vec{x}_2 - \vec{x}_1\| \|\vec{x}_2 - \vec{x}\|}$$

$$\hat{\phi}(\vec{x}) = \frac{(\vec{x}_1 - \vec{x}) \times (\vec{x}_2 - \vec{x})}{\|(\vec{x}_1 - \vec{x}) \times (\vec{x}_2 - \vec{x})\|}.$$

Integration
RK4 solves EoM

$$\frac{d\vec{v}}{dt} = \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B})$$

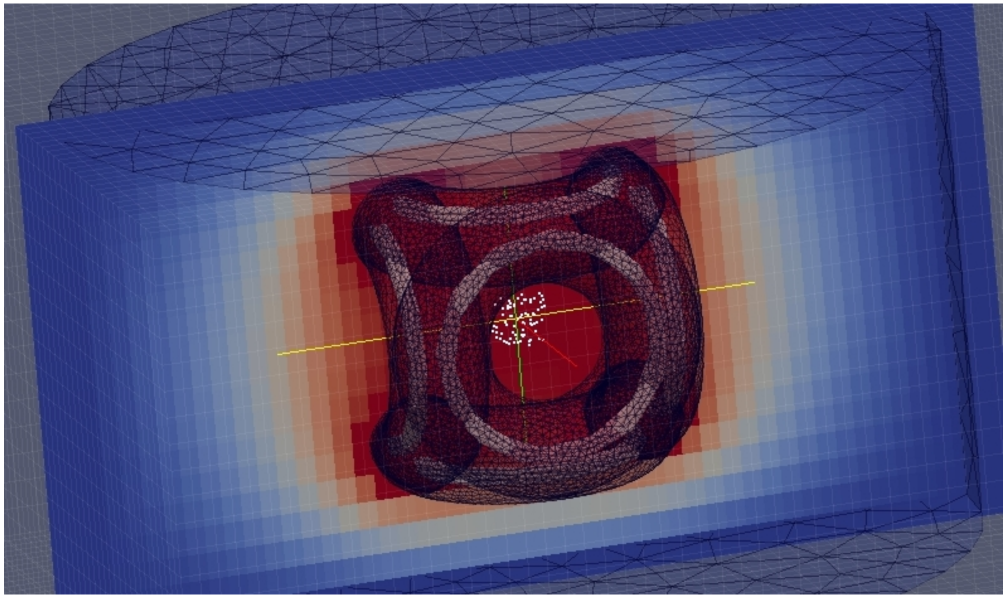
$$\frac{d\vec{x}}{dt} = \vec{v}.$$

Trilinear Interpolation: The B-field due to the wires is saved on a Cartesian grid and the interpolant is used to evaluate RHS of ODE system for RK4. This makes field definitions from arbitrarily complex coils equally inexpensive.

SSUBPIC code implementation

Complex geometry

- Triangle (STL) mesh generated by free Gmsh software, or any CAD package
- Cartesian cells intersecting triangles marked for constant (Dirichlet) BCs
- Gmsh can also make line meshes for coil windings



Poisson field solution

- Standard 2nd order central

$$
\begin{aligned}
&(\Delta x)^{-2}(\phi_{i+1,j,k} - 2\phi_{i,j,k} + \phi_{i-1,j,k}) + \\
&(\Delta y)^{-2}(\phi_{i,j+1,k} - 2\phi_{i,j,k} + \phi_{i,j-1,k}) + \\
&(\Delta z)^{-2}(\phi_{i,j,k+1} - 2\phi_{i,j,k} + \phi_{i,j,k-1}) = S_{ij}.
\end{aligned}
$$

- Solve linear system in parallel (OpenMP,MPI) with fast library (Lis - Library of Iterative Solvers)

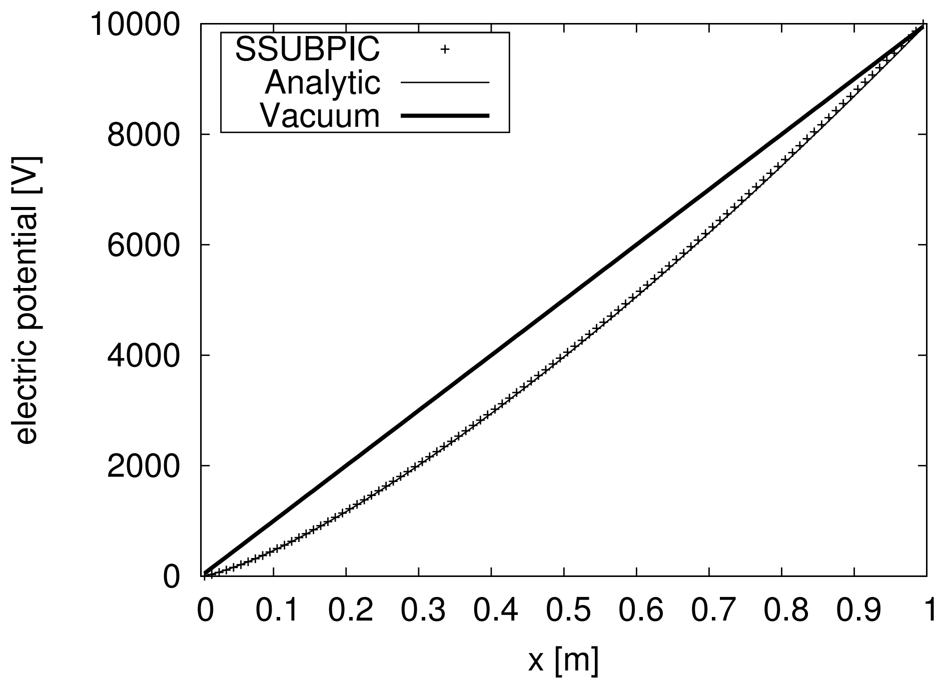
Example: Unstructured boundary definition defining stair-steps in structured code.
Note: This geometry with heavy overlap not used later!

Test Cases: 1D Child-Langmuir

- Code operated in 1D mode
- Standard Child-Langmuir space charge limited current problem
- d=1 m gap, V_0=10kV, electrons
- Theory predicts maximum current and corresponding potential profile

$$J_{CL} = \frac{4\epsilon_0}{9}\sqrt{\frac{2e}{m}}\frac{V_g^{3/2}}{d^2}, \quad V = V_0\left(\frac{x}{d}\right)^{4/3}$$

- J started below theoretical J_{CL} and incremented up until convergence fails. J_{CL} overpredicted by 9%, 5%, and 2% for 100, 200, and 400 grid points (1st order accuracy).



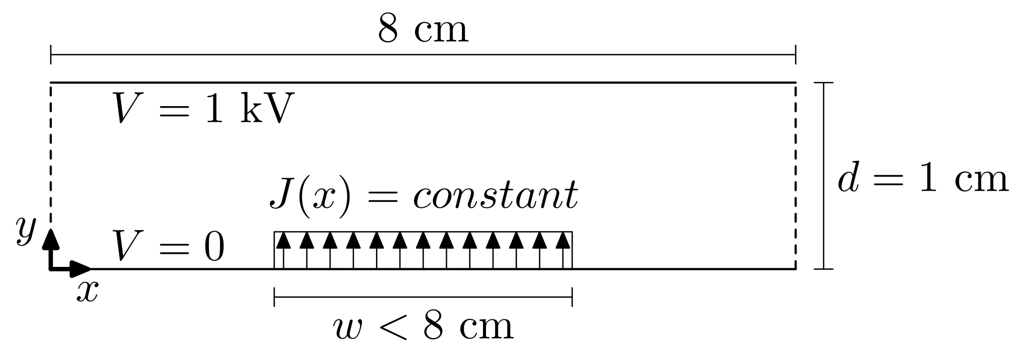
Potential profile for 100 cells in x-direction. Electrons originate from cathode at left and fly to the right. In the space-charge limited case E and hence dV/dx approach zero at the cathode to prevent further electron inflow.

Test Cases: 2D Child-Langmuir

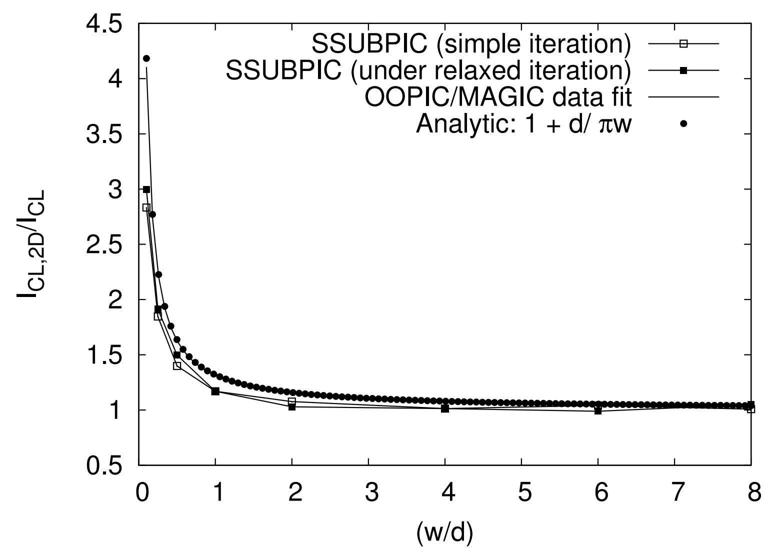
- Code operated in 2D mode
- Finite width ($= w$) patch emits electron current.
- Finite patch emits electron current.
- Prior OOPIC and MAGIC code solution by Luginsland et al [LLG96].
- Later analytic solution by Lau [Lau01].

$$\frac{J_{CL,2D}}{J_{CL}} = 1 + \frac{d}{\pi w}$$

- Again J started below theory and walked upwards until convergence failure. SSUBPIC fails below theory (1600x400 mesh, $\Delta t = 1E - 11$, 3200 e^{-}'s). Believe issue relates to interpolation scheme in first cell (see Watrous et al [WLF01])



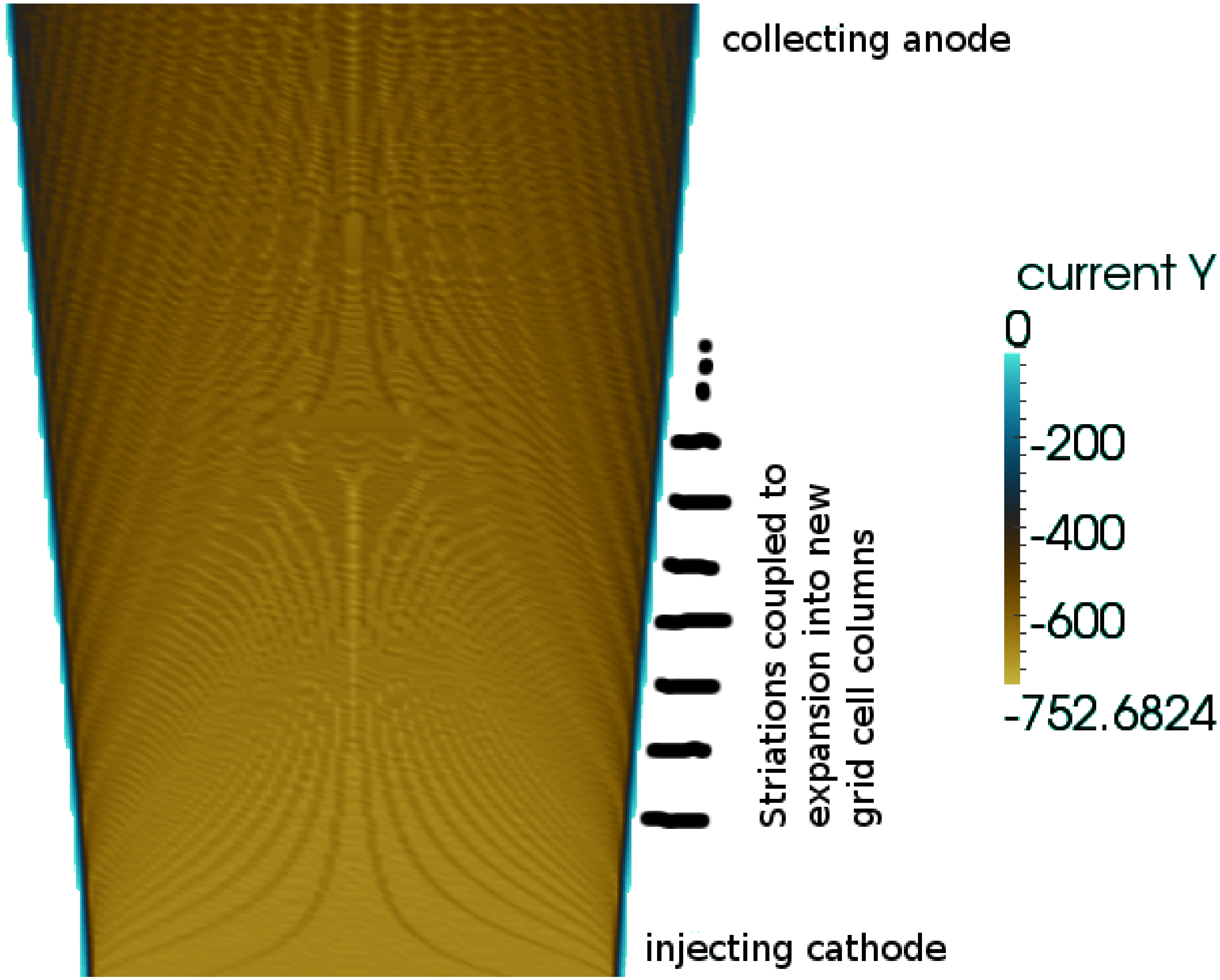
Schematic of simulation.



Under-relaxed iteration refers to averaging fields from two prior sets of simulation particles.

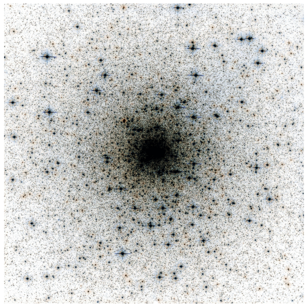
Test Cases: 2D Child-Langmuir

- Stable solutions at high current can exhibit unphysical striations that are related to grid spacing.
- This is not good, but can be watched for.

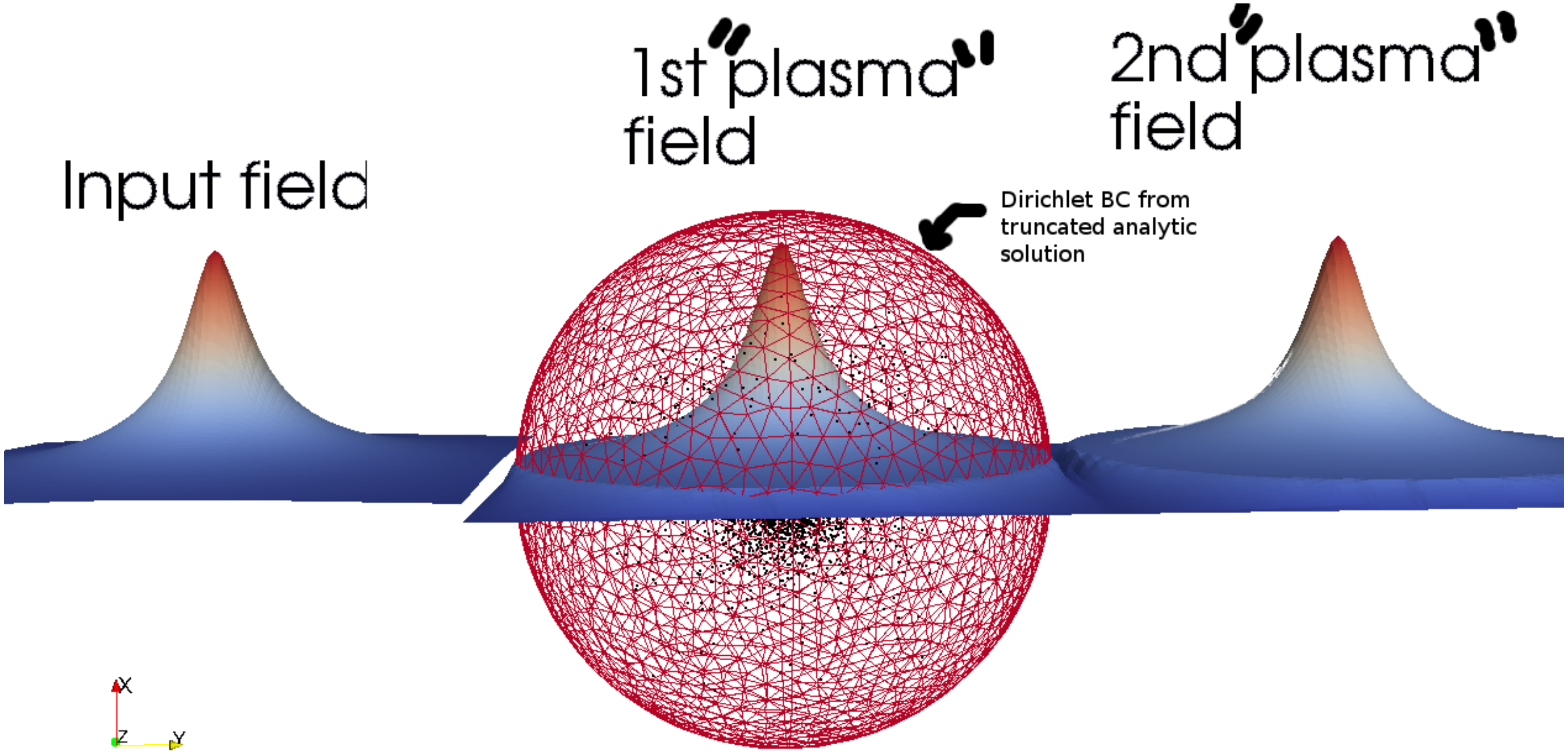


Test Cases: 3D spherical galaxy/cluster

- Stars also obey a collisionless Boltzmann equation (CBE) with a single self-attracting species. SSUBPIC in 3D mode can maintain a standard Plummer model (see Wikipedia) equilibrium globular cluster if initialized with analytic phase-space profile.



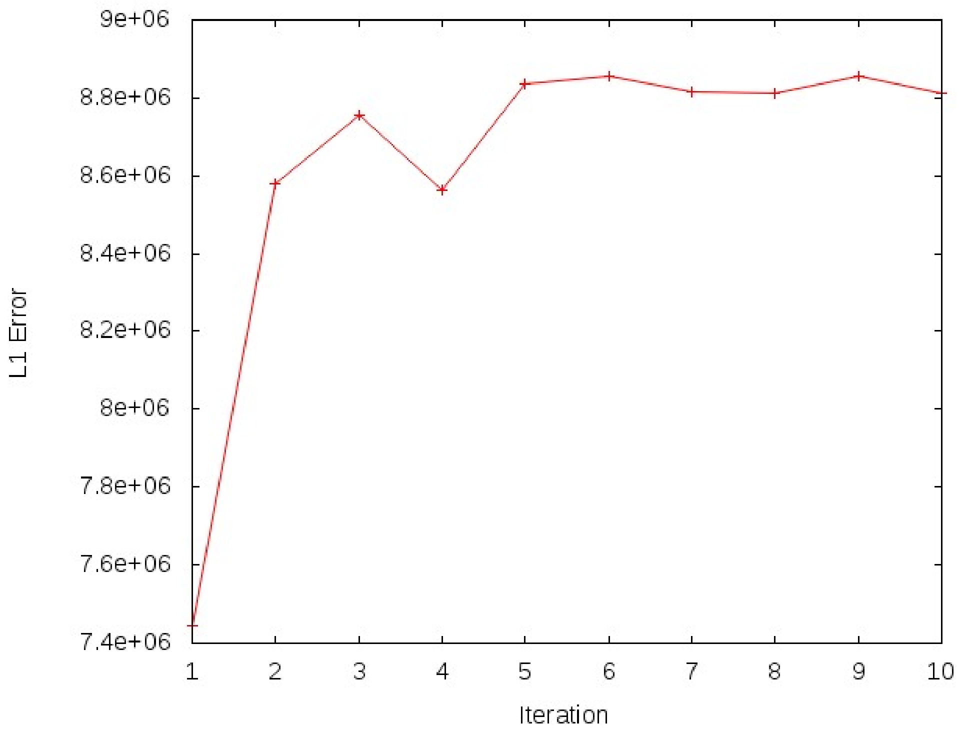
M15 - typical globular cluster



Analytic gravitational potential field given as input. Particles integrated for a set time reproduce their input field again and again.

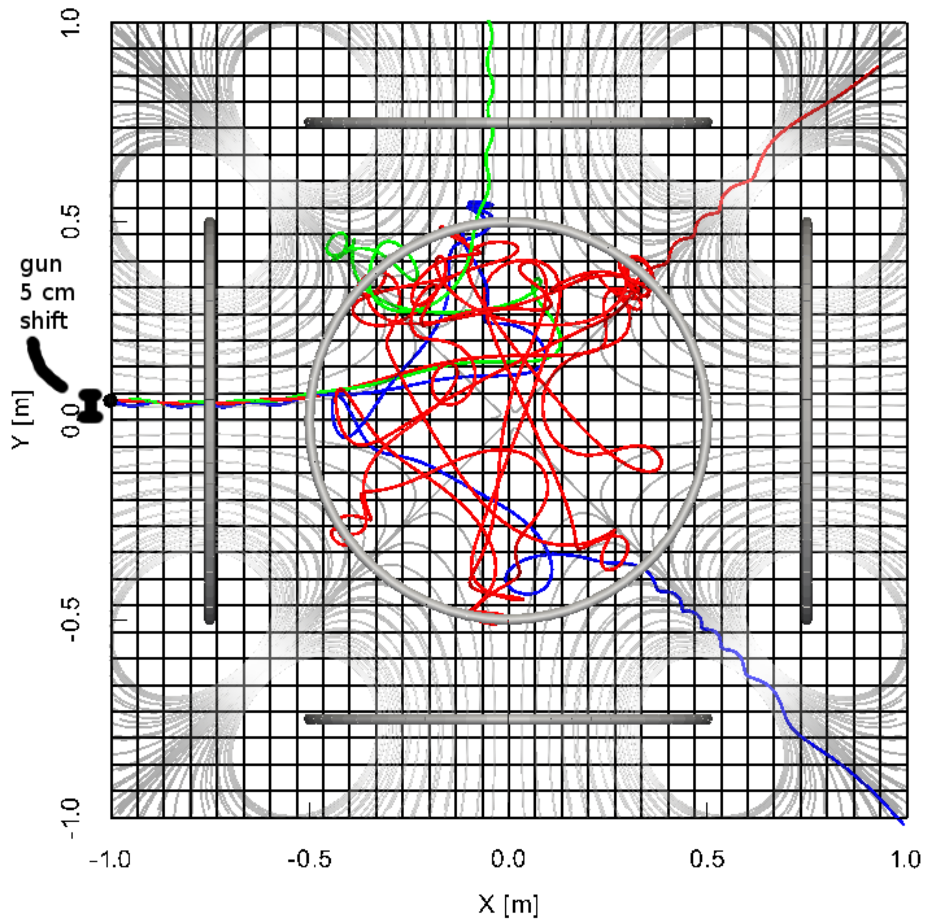
Test Cases: 3D spherical galaxy/cluster

- Error in gravitational potential settles to constant value after certain number of iterations.
- Will be higher for longer simulation time or longer time-step as RK4 does not conserve energy.
- Note: Case does not have sources and sinks as with plasma emission cases.



Application: Low-density polywell

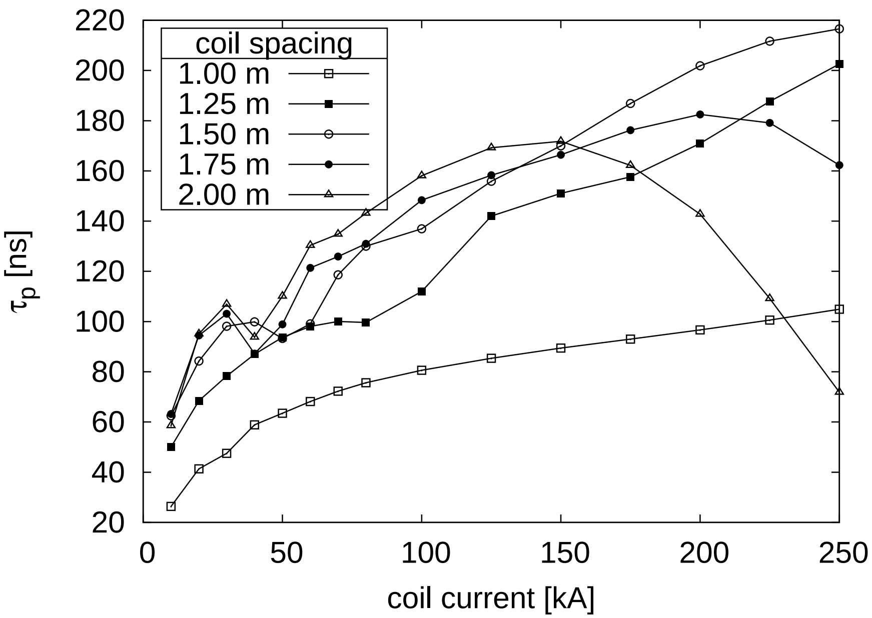
- Polywell test case without self-fields (i.e. one outer iteration only).
- Coil radius: 1 m, spacing 1.5 m, domain 2 m cube
- 30 kA current in each loop
- Electrons sourced at 20 keV ($T_e = 1keV$) on walls; 5 cm upward shift.
- 360 straight wire segments used to make six round coils
- No structures. Electrons lost when they exit cube domain.
- Particle confined for time $\tau_p = 98$ns (only 4x better than $B = 0$!)
- Comparable to bad confinement reported in Matt Carr thesis



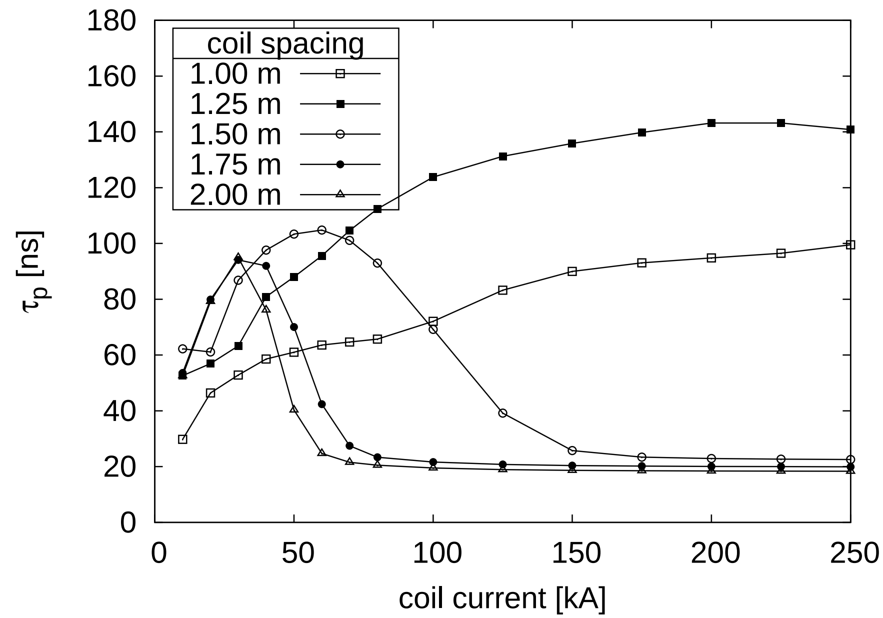
Three out of 10^6 example orbits shown.

Application: Low-density polywell

Confinement times are strongly dependent on coil current (i.e. B) and offset distance of guns; but are sub-microsecond for practical parameters.



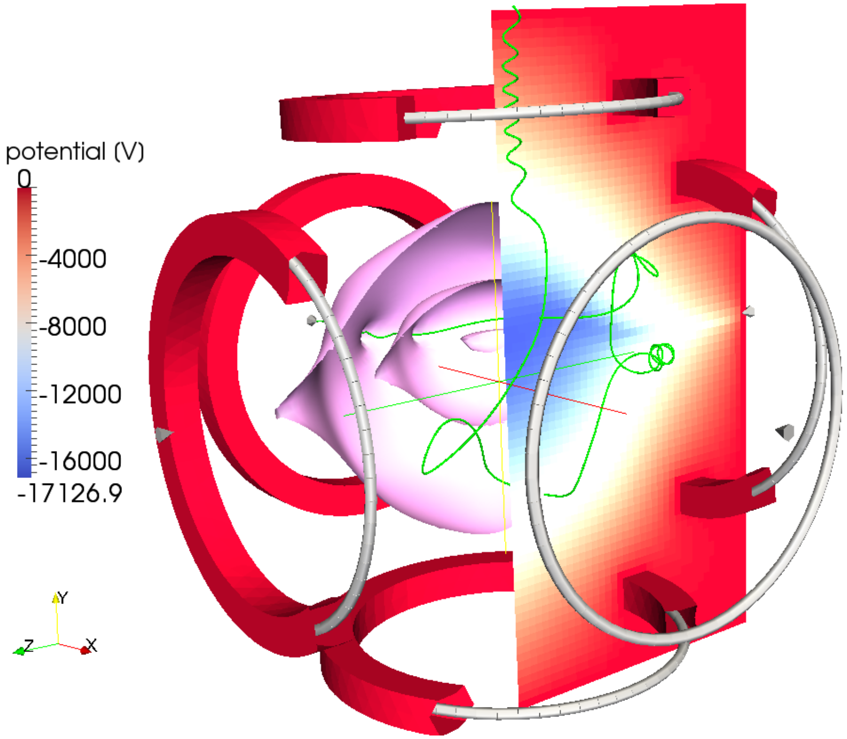
(a) 5 cm gun offset



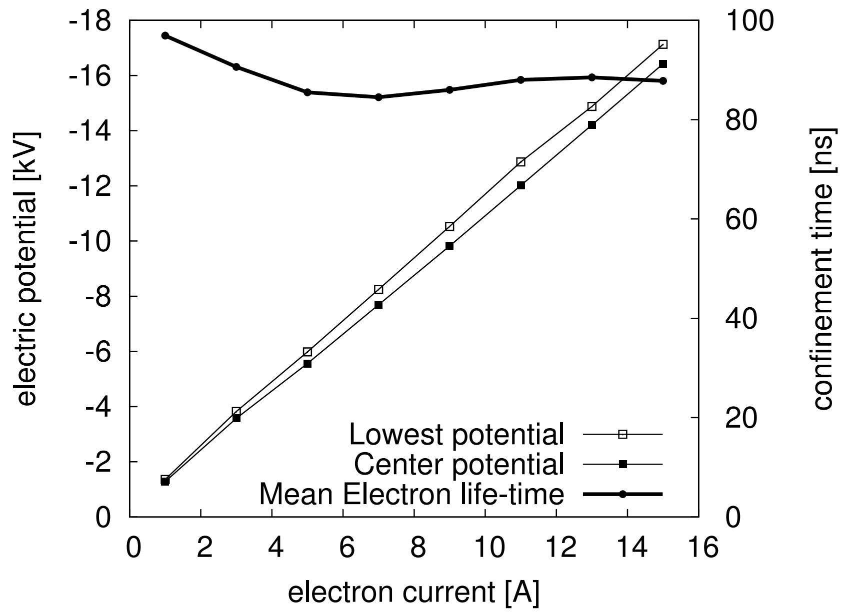
(b) 10 cm gun offset

Application: Polywell with virtual cathode (ejecting e^{-}'s)

Similar geometry as two slides ago but self-induced fields are computed. Four guns offset 5 cm upwards to avoid electrons passing right through. Now electron gun current varied from 1 to 15 A. Ejecting potential hill on order of gun energy does little to reduce confinement time!



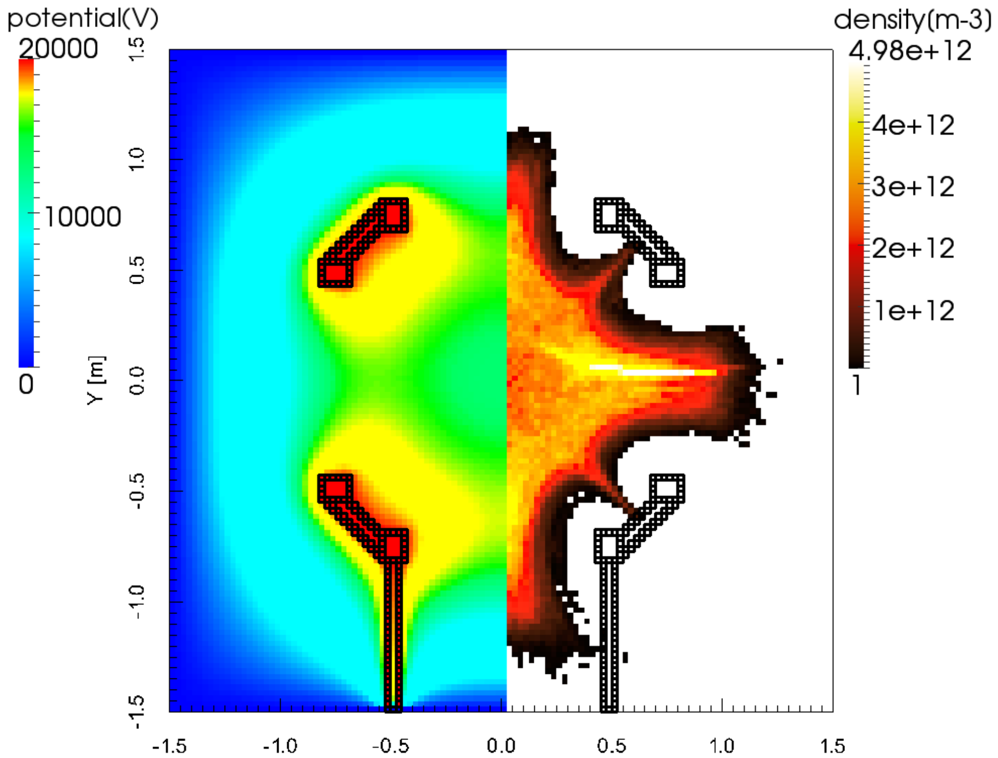
(a) Example orbit and potential contours. Coils (10 cm square cross-section) are grounded for field solutions.



(b) Well depth and confinement time as function of total e-gun current.

Application: Polywell now with charged coils (MaGrid)

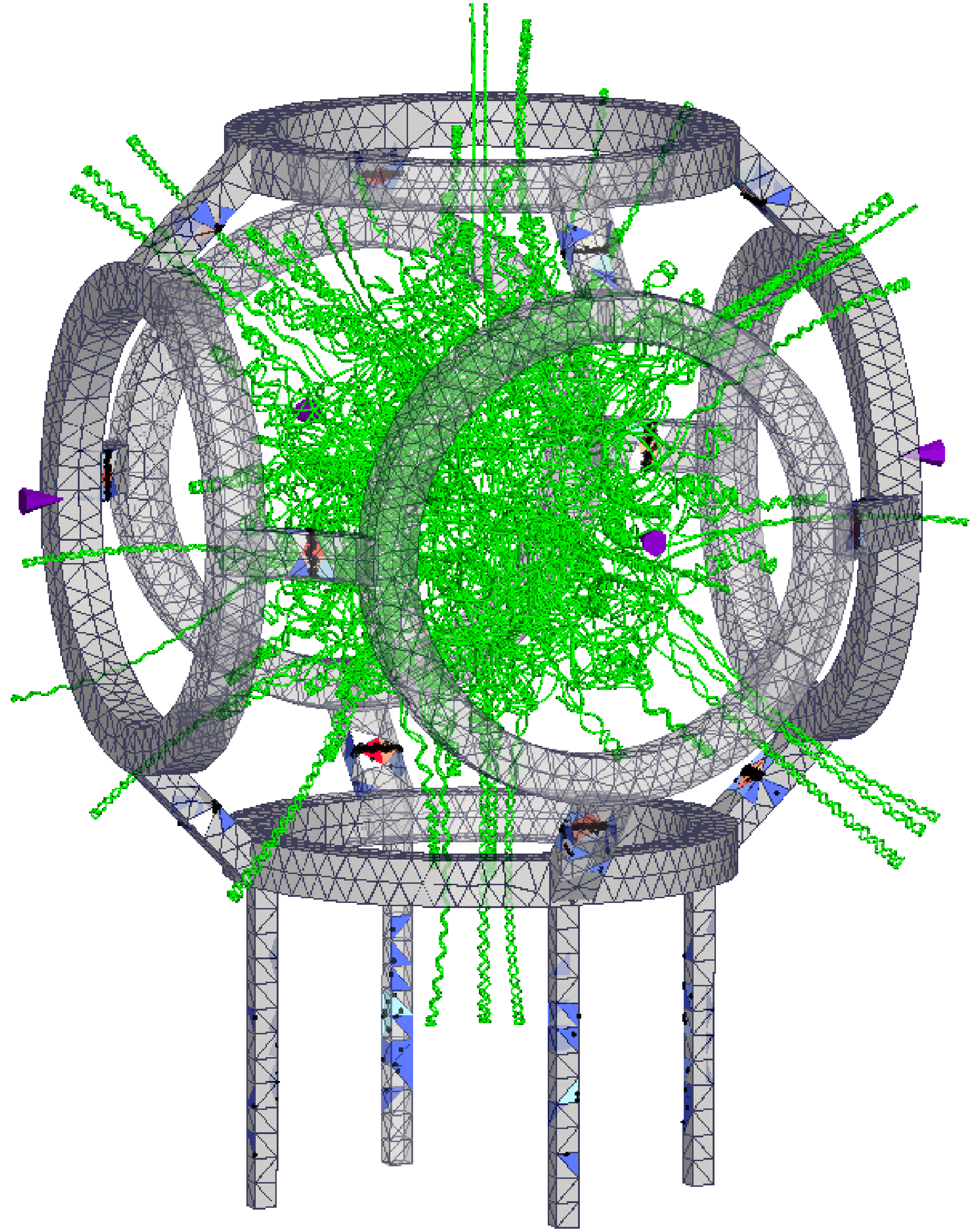
- Same geometry again, now with 3 m box (to give electrons recirculation room) and coils charged to 20 kV to draw in electrons.
- Now with no gun acceleration but still at $T_{e0} = 1keV$
- 30 kA current in each loop again
- Electrons sourced at 20 keV ($T_e = 1keV$) 2 m from center; 5 cm upward shift.
- Coils and supports have 10 cm square cross sections; 5 cm for stilts.
- Confinement time increases to 7 μs due to recirculation.
- Converged fields in five iterations ($\sim$ 10 minutes CPU time)



Electron number density and electric potential in converged solution.

Application: Polywell now with charged coils (MaGrid)

- Only 1000 particles per iteration needed for converged results (one orbit shown here).
- Black dots show where particles leave domain by hitting triangle boundaries.
- All particles leave simulation on supports without magnetic shielding (internal current carrier)
- Confinement time in recirculation system limited only by shielding quality or onset of instabilities.



Conclusions & Future Work

- Steady-state PIC is FAST. 6D computations are performed in 5-10 CPU minutes with SSUBPIC.
- Charging Polywell coils to positive bias is an effective way to increase single species confinement.
- Confinement is limited dominantly by structures without magnetic shielding.
- Numerical technique is extensible to collisional regime via Monte Carlo algorithms (Nanbu, Takizuka-Abe, etc)
- It may be extensible to high density two species plasmas using quasi-neutrality condition. Will work with gyrocenter tracking.
- No transient information (e.g. anomalous transport) unless modeled as false collision operator in which case the Reynolds-averaged kinetic equation is being solved. Krall & Rosenthal developed such a false collision operator in a time-dependent PIC code [KR95, KR91].

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