Simulations for Multiple-grid IEC

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VIEW FROM MY HOTEL ROOM

University of Maryland Flag!
Multiple-grid IEC – brief history

Sedwick et al. used additional grids to focus ion beams and increase ion confinement time.

3 TAKEAWAYS:

1: **Ion lifetimes extended:** From 10’s of passes to $10^3$-$10^6$ passes

   Greater confinement time
   + Counter-stream instability
   + IEC trap kinematics
   = Ion bunching

2: **Bunch synchronization → Decreased thermalization**
Multiple-grid IEC – current research

DODECAHEDRAL GRIDS

• 12 Faces → 6 beamlines
• Highly symmetric
• Another possibility: Truncated Icosahedron (Soccer Ball)
• Feed-throughs?

ION BUNCHING

• Potential well can be shaped to encourage ion bunch cohesion

MAGNETIC CORE

• Confinement of electrons in the core
2-GRID IEC
2-GRID IEC

Electric potential along beamline

Potential (kV) vs Distance (m)

-70 -60 -50 -40 -30 -20 -10 0 10 20 30 40 50 60 70

-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1
4-GRID IEC
4-GRID IEC

Electric potential along beamline

ION CONFINEMENT

ELECTRON CONFINEMENT

Potential (kV)

Distance (m)

Background / Particle-particle model / Hybrid PIC model / Conclusion |
Particle-particle Discrete Event Simulation

- **Inter-particle forces are calculated directly** (N-body simulation)
  - No need to solve Poisson’s equation at each time step

- **No global time-step**, each particle is assigned its own time-step depending on its velocity and acceleration
  - Coulomb collisions are modeled directly by decreasing the time-step values of colliding particles.

- Static E&M fields are calculated once at the beginning of the simulation

Mathematical equation:
\[ \vec{a}_i = -\frac{1}{4\pi \varepsilon_0} \sum_{j \neq i} \frac{q_j}{m_i r_{ij}^3} \vec{r}_{ij} + \frac{q_i}{m_i} \vec{E} \]

Diagram:
- P1, P2, P3
- Interactions between particles
- Queue tracking
2-GRID Particle-particle simulation

Grid size: 201-by-201-by-201
Elapsed simulation time: 0
Elapsed computation time: 00:00:00

IONS
Active: 2000
Hit grid: 0
Left domain: 0
Mean(Δt) = 0
Macroparticle weight = 3447709

LOW DENSITY BUNCH

Ion density in xy-plane
Potential from ions
Total potential in xy
What happens if we increase the density of the ion bunch?
2-GRID Particle-particle simulation

Grid size: 201-by-201-by-201
Elapsed simulation time: 0
Elapsed computation time: 00:00:03

IONS
Active: 2000
Hit grid: 0
Left domain: 0
Mean(x,t) = 0
Macroparticle weight = 9193892

HIGHER DENSITY BUNCH
4-GRID Particle-particle simulation

Grid size: 201-by-201-by-201
Elapsed simulation time: 0
Elapsed computation time: 00:00:04

IONS
Active: 2000
Hit grid: 0
Left domain: 0
Mean($\lambda t$) = 0
Macro particle weight = 3678093

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Background

Particle-particle model

Hybrid PIC model

Conclusion

SAME DENSITY AS PREVIOUS SLIDE
Ion Bunching – The Kinematic Criterion

Ions near the back of the bunch are **decelerated** by the Coulomb repulsion from the bunch. Energy **decreases**. Period must also **decrease** to prevent ions from “running away.”

**Kinematic criterion:**

\[
\frac{dT}{dE} \geq 0
\]

**T**: Period  
**E**: Ion Energy (KE+PE)

Ions in the front of the bunch are **accelerated** by the Coulomb repulsion from the bunch. Energy **increases**. Period must also **increase** to prevent ions from “running away.”

**IONS OF DIFFERENT ENERGIES HAVE DIFFERENT PERIODS**

**Multi-grid potential well**

**TRAILING IONS**  
**LEADING IONS**
Ion Bunching – The Kinematic Criterion

Kinematic criterion: \( \frac{dT}{dE} \geq 0 \)

But \( \frac{dT}{dE} \) can’t be too large either!

Conditions have to be just right for ions to coalesce into bunches

**FUTURE WORK:**
“Sculpting” the IEC well to encourage bunch cohesion
Electron confinement in the IEC core
Electron confinement in the IEC core

2D Analogue

Cathode grid

Inner anode grid

Outer anode grids
Electron confinement in the IEC core

- Electron prevented from escaping along beamline by electric field
- Electron prevented hitting anode grid by magnetic mirror

Cathode grid
Inner anode grid
Outer anode grids
Electron confinement in the IEC core
Approximate E&M fields along a beampath
Confinement of a single electron
Confinement of many electrons

Grid size: 201-by-201-by-201
Elapsed simulation time: 0
Elapsed computation time: 00:00:00

**ELECTRONS**
Active: 2400
Hit grid: 0
Left domain: 0
Mean(Δt) = 0
Macroparticle weight = 383135

- Electron density in xy-plane
- Potential from electrons
- Total potential in xy
Disadvantages of the particle-particle discrete event simulation

- Computation time scales as $N^2$
- Only suitable (at this point) for modeling one species at a time (ions or electrons) for short timescales
- To model both species at once we need a hybrid PIC model
Hybrid Particle-in-cell model

IONS $\rightarrow$ PARTICLES

ELECTRONS $\rightarrow$ FLUID

Long-timescale simulation requires time-steps based on the ion motion

Assume electrons reach a thermalized steady-state at each time-step \( \frac{dn_e}{dt} = 0 \)
Currently using the following governing equations:

**CONTINUITY**

\[ \frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{v}) = S \]

**VELOCITY**

\[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = \frac{e}{m_e} (\nabla \Phi + \vec{v} \times \vec{B}) \]

**POISSON’S**

\[ \nabla^2 \Phi = \frac{e}{\varepsilon_0} (n_e - n_i) \]

Interested in steady-state solution

But first we’ll test the time-stepping solution
Electron fluid model – TEST PROBLEM (not IEC)

Wire locations (perpendicular to plain)

Electron source (#/m³/s)

B-field direction

B-field magnitude (T)
Electron fluid model

Simple 2D test problem:
- Magnetic field created by current-carrying wires
- Electron Source in Center
Electron fluid model

Simple 2D test problem:
- Magnetic field created by current-carrying wires
- Electron Source in Center

Comparison with particle-in-cell model with same conditions
Electron fluid model

Simple 2D test problem:
- Magnetic field created by current-carrying wires
- Electron Source in Center

Comparison with particle-in-cell model with same conditions

FLUID MODEL

PIC MODEL
Electron fluid model

With increased B-field
Summary

Pathway to net power fusion

ION BE FOCUSING

Reduce ion–ion collisions to the device core
Reduce ion–grid collisions

REDUCE THERMALIZATION

REDUCE BREHMSSTRAHLUNG LOSSES

DIRECT ENERGY CONVERSION

Limit ion–ion collisions to the device core
Reduce ion–grid collisions

Monoenergetic ions
Low energy operation (~135 keV)

REDUCE BREHMSSTRAHLUNG LOSSES

DIRECT ENERGY CONVERSION

p-11B fuel

Background
Particle-particle model
Hybrid PIC model
Conclusion

Proton

Helium

Inuclearfusion.com

ION BE FOCUSING

Reduce ion–ion collisions to the device core
Reduce ion–grid collisions

REDUCE THERMALIZATION

REDUCE BREHMSSTRAHLUNG LOSSES

DIRECT ENERGY CONVERSION

p-11B fuel

Helium

Inuclearfusion.com
The End

Grid size: 201-by-201-by-201
Elapsed simulation time: 0
Elapsed computation time: 00:00:22

IONS
Active: 12000
Hit grid: 0
Left domain: 0
Mean(Δt) = 0
Macro-particle weight = 766259
**DENSITY**

\[
\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (nv) = S
\]

**VELOCITY**

\[
\frac{\partial v}{\partial t} + \frac{\partial}{\partial x} (v^2) = \frac{e}{m} \frac{\partial}{\partial x} \Phi
\]

**POTENTIAL**

\[
\frac{\partial^2}{\partial x^2} \Phi = \frac{e}{\epsilon_0} n
\]

Using the **Roe scheme** (basically a selective upwind scheme) for density and velocity:

\[
\frac{\partial u_i}{\partial t} + \frac{1}{2\Delta x} \left\{ (\Gamma_{i+1} - \Gamma_{i-1}) - \left| \frac{\Gamma_{i+1} - \Gamma_i}{u_{i+1} - u_i} \right| (u_{i+1} - u_i) + \left| \frac{\Gamma_i - \Gamma_{i-1}}{u_i - u_{i-1}} \right| (u_i - u_{i-1}) \right\} = S_i
\]

- Central differencing
- Upwind correcting terms
For STEADY STATE: can’t take the derivatives of absolute values:

\[
\frac{\partial u_i}{\partial t} + \frac{1}{2\Delta x} \left\{ \left( \Gamma_{i+1} - \Gamma_{i-1} \right) - \frac{\Gamma_{i+1} - \Gamma_i}{u_{i+1} - u_i} (u_{i+1} - u_i) + \frac{\Gamma_i - \Gamma_{i-1}}{u_i - u_{i-1}} (u_i - u_{i-1}) \right\} = S_i
\]

Use approximation: \( |x| \approx \sqrt{x^2 + a^2} \) with small enough \( a \)

In our case, \( a \) is a velocity