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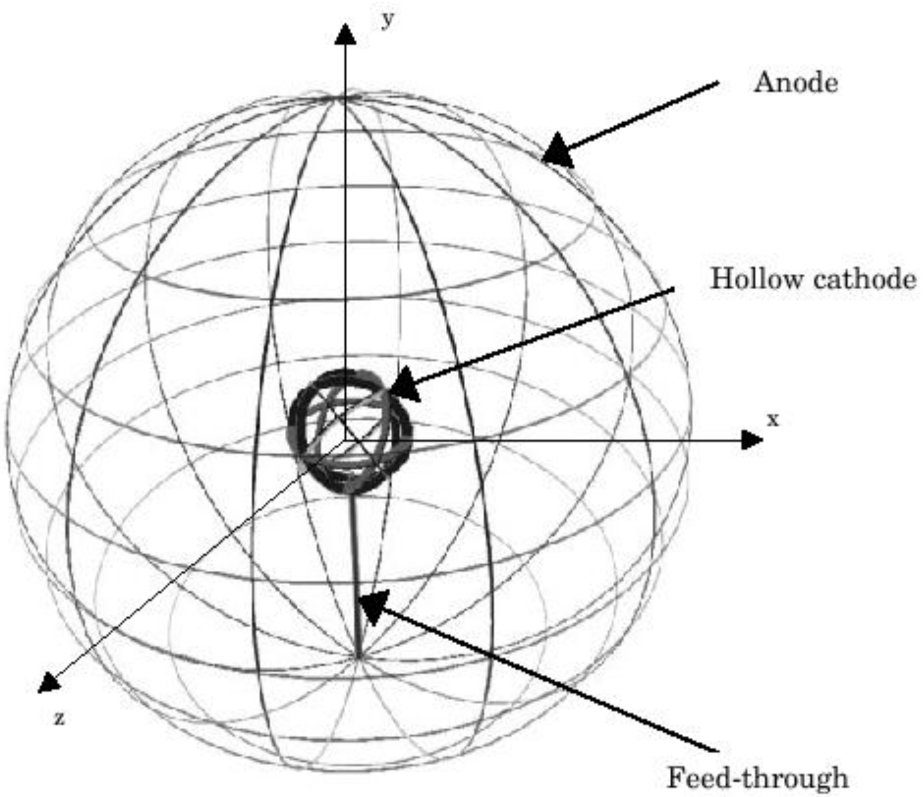
Orbit theory study of electron confinement in a Polywell™ device

David Gummersall |PhD Student
Supervised by Dr Joe Khachan

Image: R. W. Bussard, 57th International Astronautical Congress, 2006

Gridded cathode IEC

Traditional gridded cathode IEC device:



Problems and limitations
for power production:

• Energy loss due to ion collisions
with the grid.

• Contamination of the plasma via
ablation and sputtering of the cathode

Polywell™ a virtual cathode system

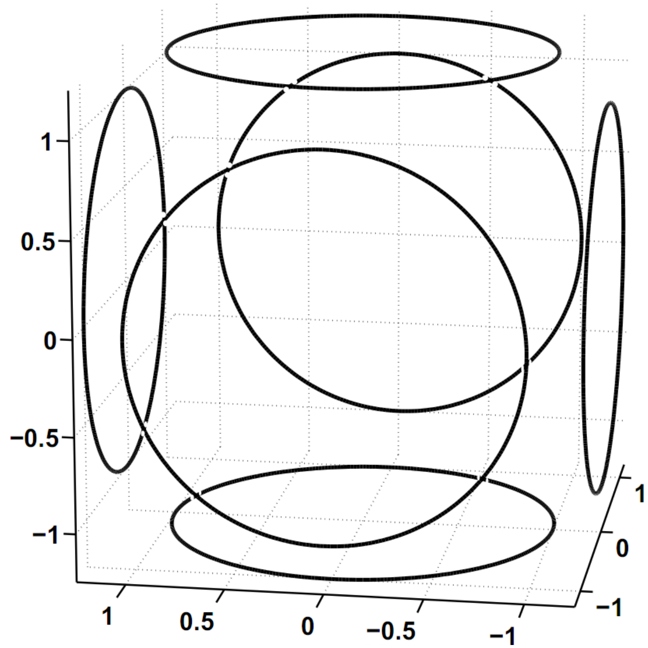


Fig. 1 A three dimensional schematic layout of Bussard's Polywell™ design.

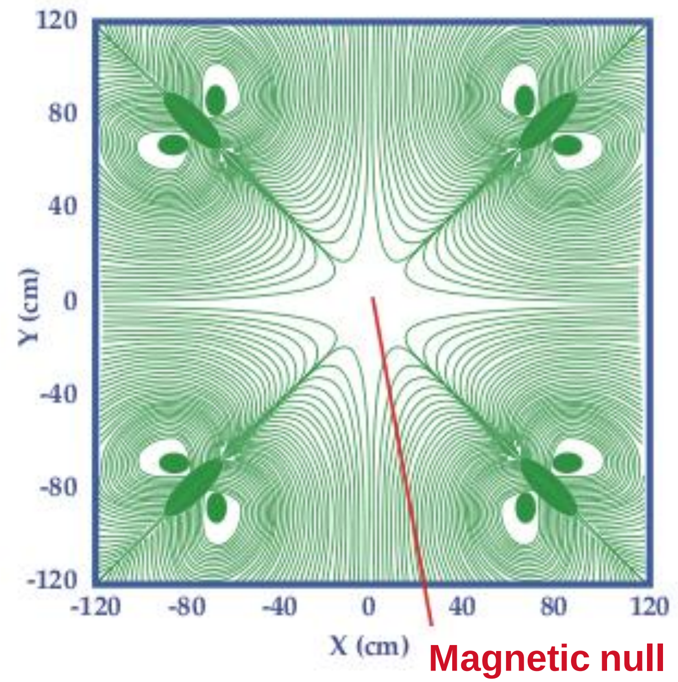


Fig. 2 Magnetic field structure inside Polywell™, highlighting the magnetic null.

- Central minimum magnetic field has certain plasma M.H.D. Stability properties.
- Virtual cathode may form due to space charge trapping of electrons.
- Ions are then electrostatically confined by the electron's electric field.

Fig 2. Image: N. Krall, Physics of Plasmas, **2**, 1995. Reformated in colour by Mark Duncan, 2007

Aims

Aims for our parameter space were three-fold:

1. Find a scaling law for the confinement time of electrons.

2. Characterize the effect of pulsing the current in the coils of the Polywell™.

3. Determine the radial distribution of electrons.

These will be achieved through empirical and theoretical analysis of simulation data obtained via an orbit theory model of electrons.

The simulation and parameter space

Five main parameters of interest:

Param.	Description
R	Radius (m)
I	Current (A)
dI/dt	Rate of ΔI (A/s)
K	Energy (eV)
s	Coil spacing (m)

2640 Total

- 420 non-interacting electron simulations per set of parameters.
- Over of 1.1 Million simulations.
- Each electron could have up to 10^5 time steps.
- Non-interacting condition: Background density < 10^{10} cm^{-3}.
- Corresponding to a pressure < 4.14 x 10^{-5} Pa at room temperature.

Vacuum field calculations

Electrostatic source free Maxwell's Equations become in the non-relativistic limit:

$$\begin{aligned} \nabla \bullet \mathbf{E} &= 0 \\ \nabla \bullet \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} \end{aligned}$$

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}$$

Coulomb gauge:

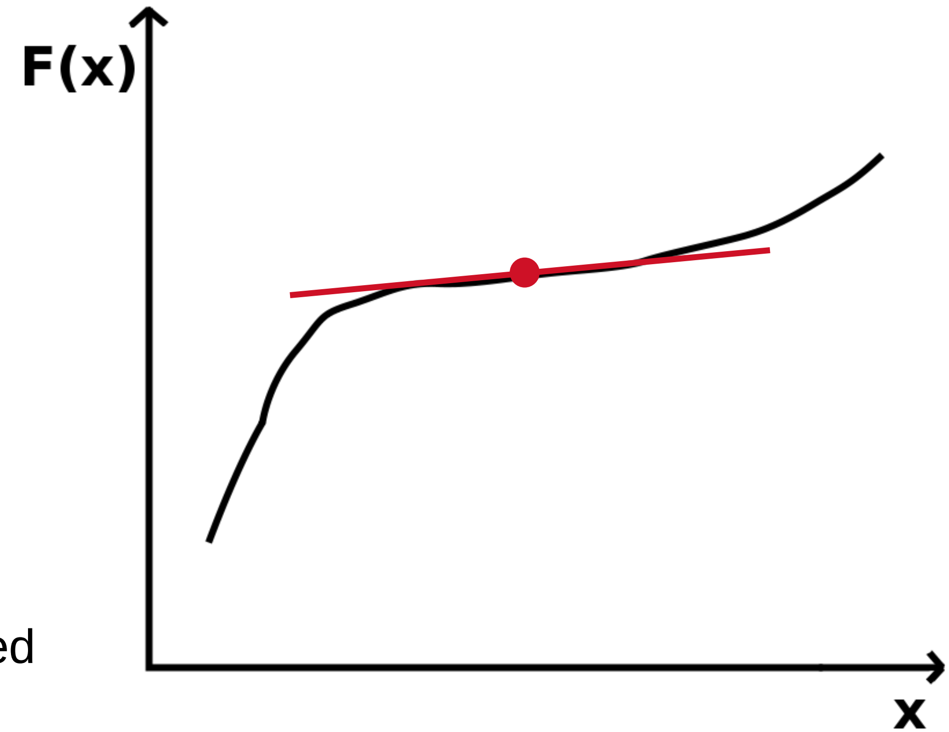
$$\nabla \bullet \mathbf{A} = 0$$

$$\mathbf{A} = \frac{\mu_0}{4\pi} I \sum_{coils} \int \frac{d\mathbf{l}}{|\mathbf{r}|}$$

E and **B** are always orthogonal: $\mathbf{E} \bullet \mathbf{B} = -\frac{\partial \mathbf{A}}{\partial t} \bullet (\nabla \times \mathbf{A}) = 0$

Equations of motion

$$m\ddot{\mathbf{x}} = \underbrace{\mathbf{F}\left(\mathbf{x}\right)}_{\text{Power series expansion}}$$



Position and velocity can be solved analytically as a function of time.

However the Lorentz force generally yields three nonlinear, coupled, second order differential equations:

$$m\ddot{\mathbf{x}} = \mathbf{F}\left(\mathbf{x}, \dot{\mathbf{x}}\right) = q\left[\mathbf{E}\left(\mathbf{x}\right) + \left(\dot{\mathbf{x}} \times \mathbf{B}\left(\mathbf{x}\right)\right)\right]$$

Equations of motion continued

The zeroth order expansion term of the Lorentz force gives constant electric (**E**) and magnetic (**B**) fields.

For constant, orthogonal electric (**E**) and magnetic (**B**) fields the solution to the equations of motion are:

$$\mathbf{x}\left(t\right)=\mathbf{x}_0+\left[r\sin\left(\phi_0+\omega_c t\right)\hat{\mathbf{E}}_\mathbf{0}-\left(r\cos\left(\phi_0+\omega_c t\right)+\frac{E}{B}\,\mathrm{t}\right)\hat{\mathbf{e}}_\mathbf{2}+v_{\parallel}t\hat{\mathbf{B}}\right]$$

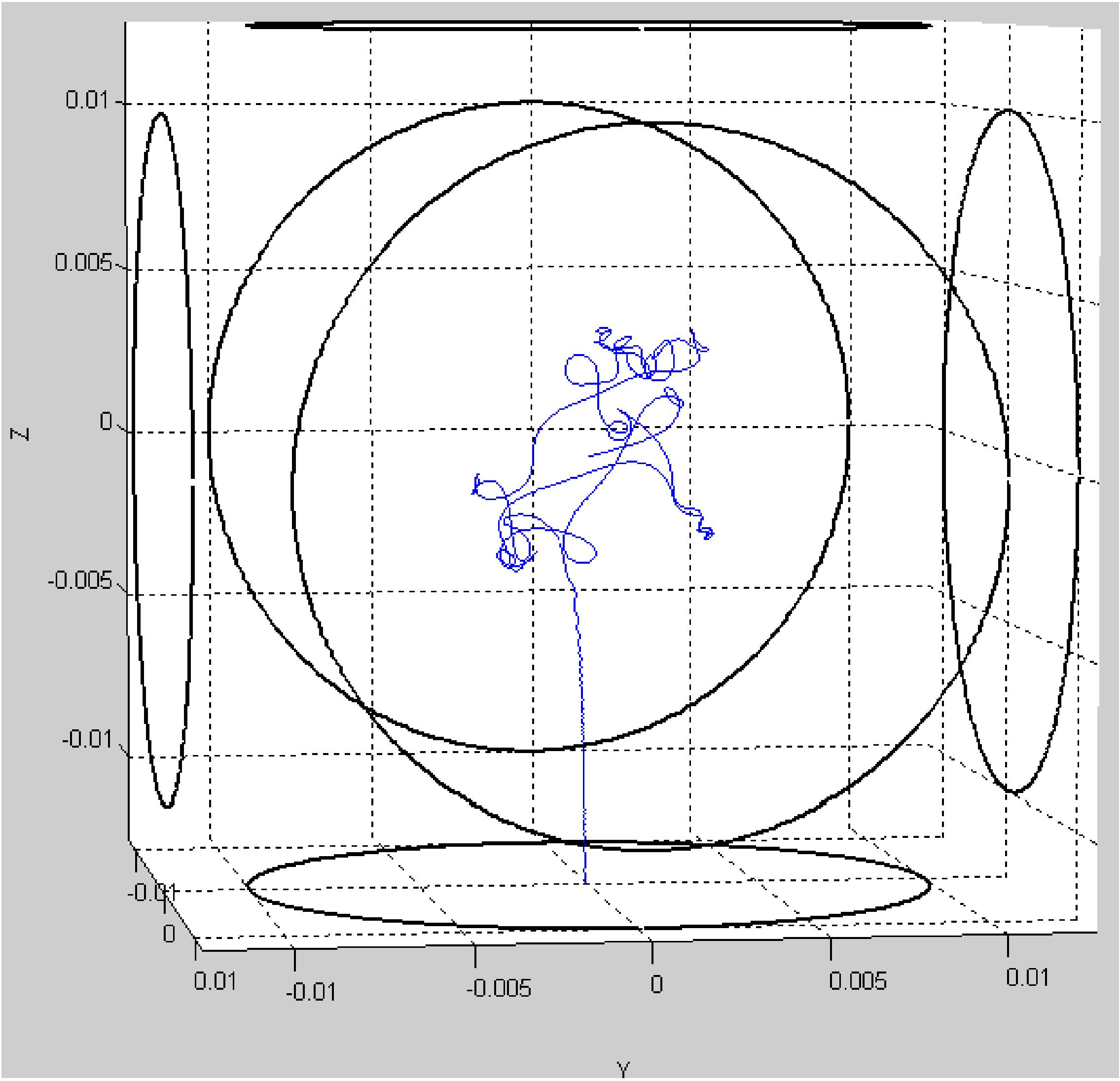
$$\dot{\mathbf{x}}\left(t\right)=\left[v_\perp\cos\left(\phi_0+\omega_c t\right)\hat{\mathbf{E}}_\mathbf{0}+\left(v_\perp\sin\left(\phi_0+\omega_c t\right)-\frac{E}{B}\right)\hat{\mathbf{e}}_\mathbf{2}+v_{\parallel}\hat{\mathbf{B}}\right]$$

r is the gyroradius radius. ω_c is the cyclotron frequency.

$$\hat{\mathbf{e}}_\mathbf{2}\equiv\hat{\mathbf{E}}_\mathbf{0}\times\hat{\mathbf{B}}$$

$\phi_0, \mathbf{x}_0$ and $v_\perp$ are determined by initial conditions.

Simulation example



R = 10 cm

I = 10 kA

Energy = 100 eV

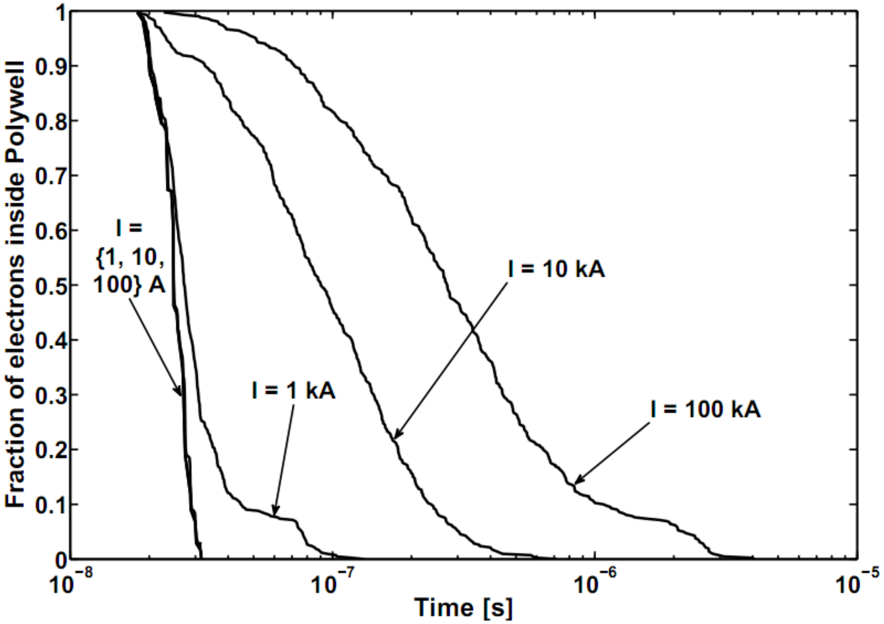


Fig. 3 Confinement time of 100 eV electrons for a 10 cm radius Polywell™.

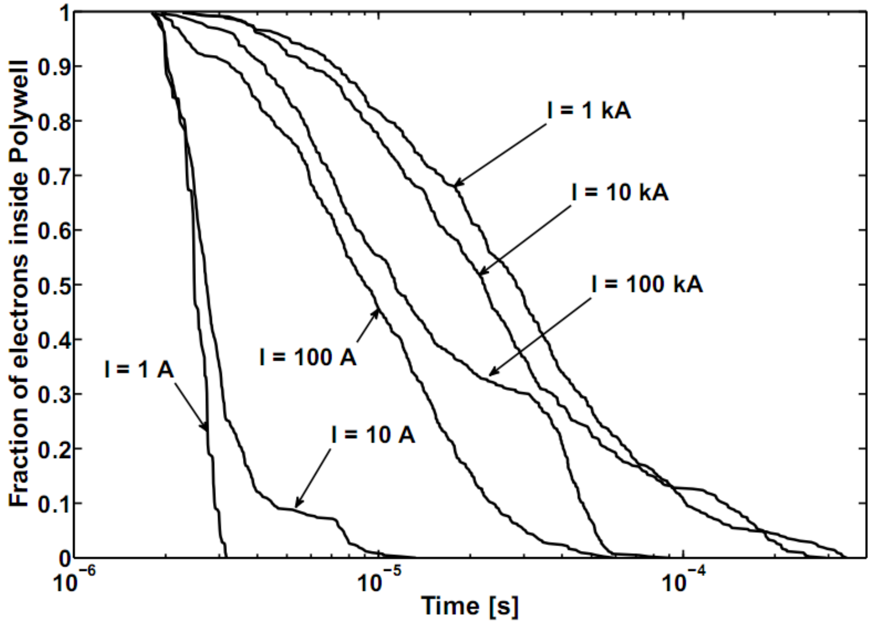


Fig. 4 Confinement time of 100 eV electrons for a 10 m radius Polywell™.

Confinement parameter window: $$1 \leq \frac{IR}{K} \leq 10^3$$

Confinement time model

Assume magnetic cusp acts like a 1D magnetic mirror.

System was adiabatically invariant a short distance from the central magnetic null.

As a result the angle between the velocity vector and magnetic field can be expressed as:

$$\theta_c = \arcsin\left(\sqrt{\frac{B_0}{B_m}}\right) \qquad \theta_c \cong \sqrt{\frac{B_0}{B_m}} \qquad B_m = B_f \times \frac{I}{R}$$

Where B_m is the maximum strength of the magnetic field and B_0 is the strength of the magnetic field in the weakest section of the adiabatically invariant path.

$$\frac{dN}{dt} = -\underbrace{\frac{\theta_c}{\frac{\pi}{2}}}_{\text{Loss probability}} \overbrace{\frac{v}{2R}}^{\text{Frequency}} N$$

$$N_0(t) = \exp\left[-\sqrt{\frac{2B_0 e}{m B_f \pi^2}} \times \sqrt{\frac{K}{IR}}\, t\right]$$

Normalized

Confinement time analysis

For R~1m:

$$N_0(t) = \exp\left[-\sqrt{\frac{2B_0 e}{mB_f\pi^2}} \times \sqrt{\frac{K}{IR}t}\right]$$

B_0 was varied to fit data subject to the condition: $B_0 < B_m$.

However a more accurate empirical model for all R was found to be:

$$N_0(t) = \exp\left[-(2.0 \pm 0.6)\,10^6\sqrt{\frac{K}{IR^3}t}\right]$$

Time constant/scaling law:

$$t_n \propto \sqrt{\frac{IR^3}{K}}$$

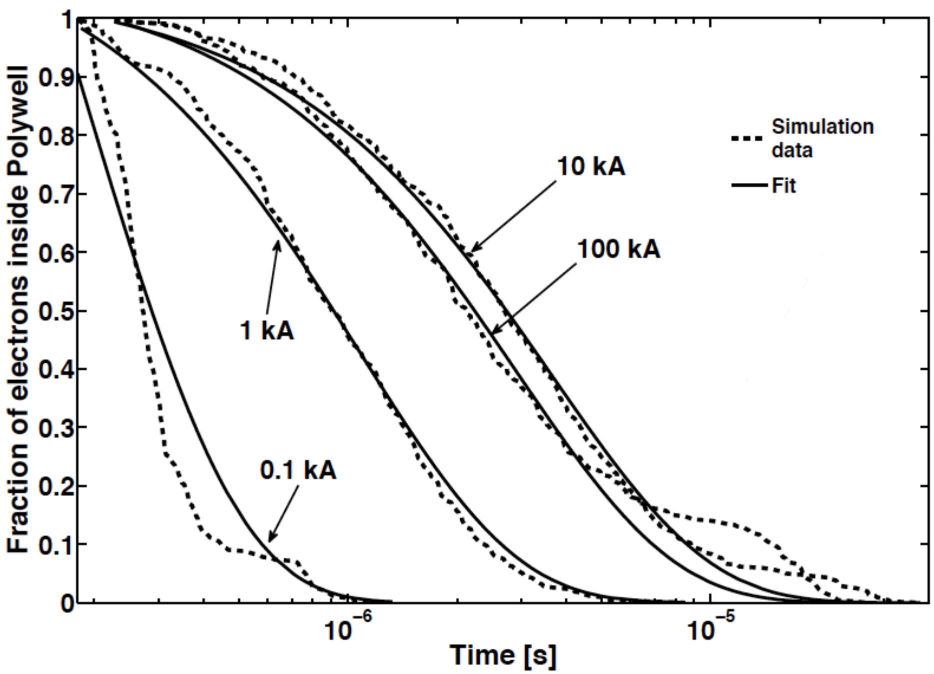


Fig. 5 A sample of fitted curves to a 1 m radius Polywell™ with 100 eV electrons where the line data were the best fit curves to corresponding data.

Pulsed current confinement time results

Can **E** x **B** drift effect confinement time?

Lower coil currents result in nearly indistinguishable curves between pulsed and steady state operation.

Negligible result within our parameter range for dI/dt.

This outcome has also been shown to be independent of R and K.

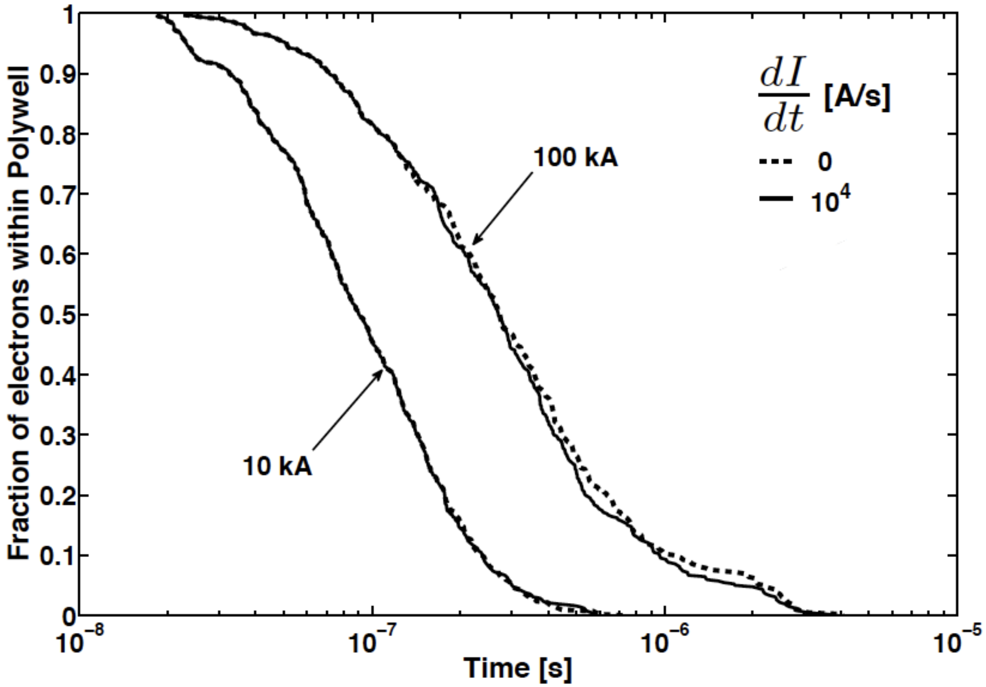


Fig. 9 Fraction of 100 eV electrons contained within the Polywell ™, with R=10 cm.

The magnetic force is proportional to, v|**B**|, will in general be of order 10^6 N greater than the electrical force in this simulation.

Work done by the **E** field

How much work is done by the Electric field?

Define: $$F = \left| \frac{K_f - K_i}{K_i} \right|$$

$$F = 4.57 \times 10^{-9} \times R \times \frac{dI}{dt}$$

F was very small for our dI/dt parameter range.

Thus steady state condition assumed:

$$\frac{dI}{dt} = 0$$

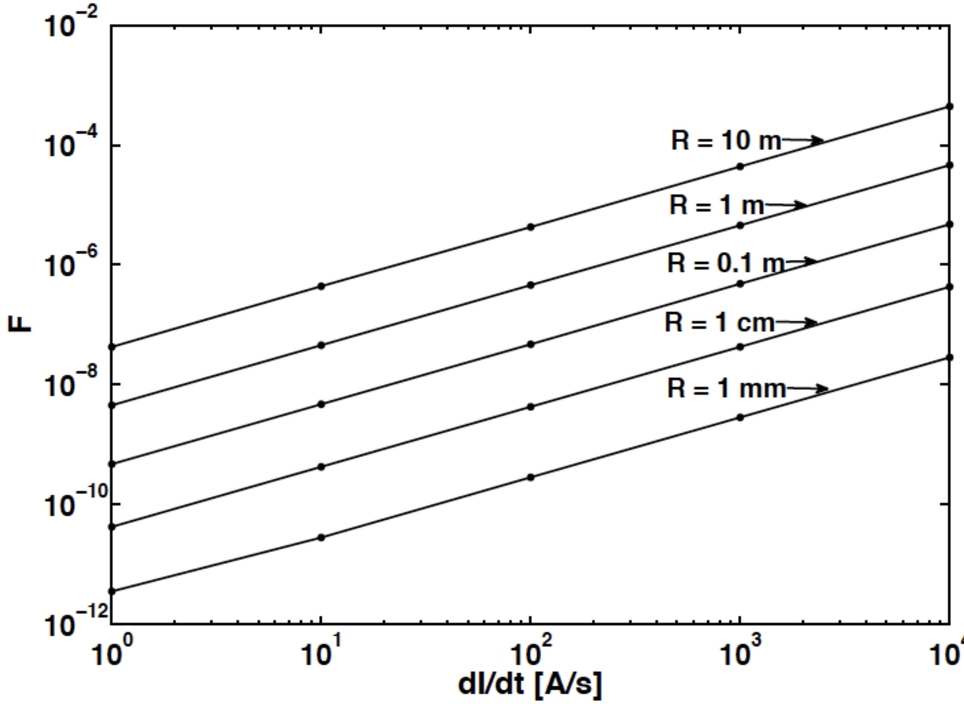


Fig 10 Comparison of average absolute fractional change in electron energy with Radii (R).

Average radius results

Time-weighted average electron radial distance from center of the Polywell™.

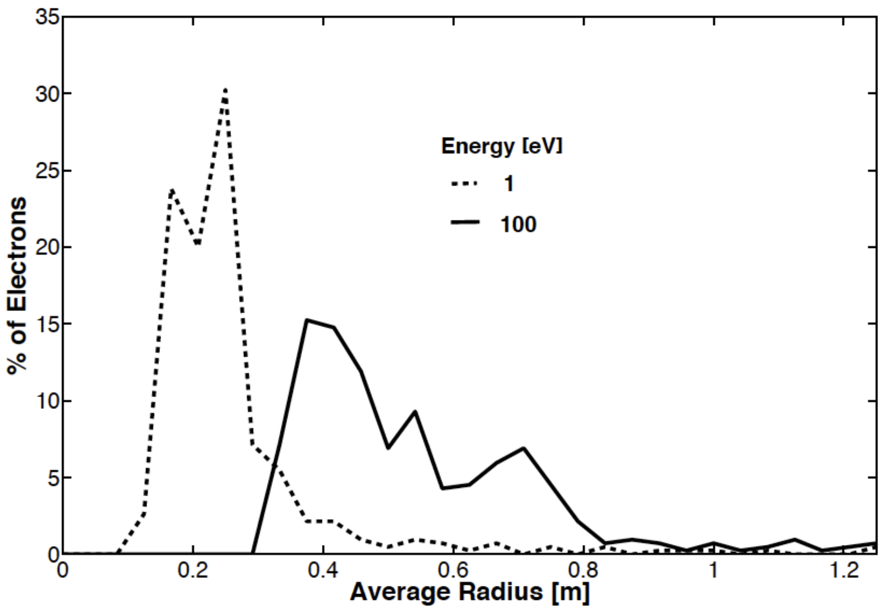


Fig. 6 A smoothed plot of radial electron localization with 100 kA current in a 1m radius Polywell™.

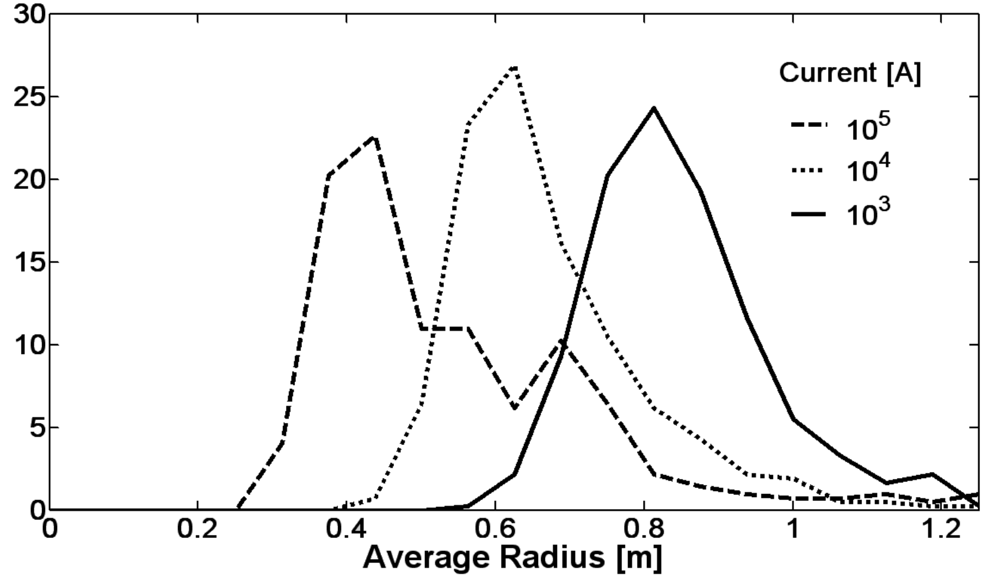


Fig. 7 Illustrating the average radial localization of 100 eV electron's for the highest three currents simulated with R = 1m.

- Radius corresponds approximately to the positions at which electrons were reflected, hence forming a 'shell' of electrons.
- Average electron radius increasing with electron energy.
- Radius decreasing with current.
- Wider distribution in localization width with increasing energy.

Average radius analysis

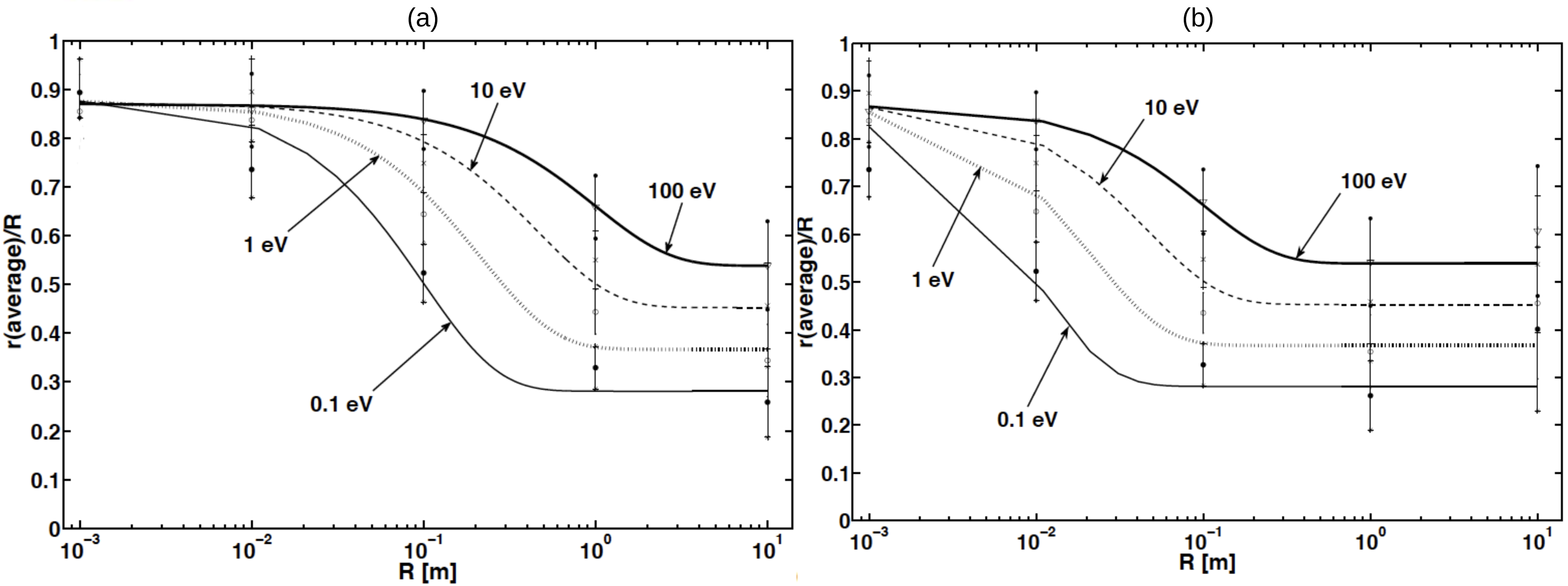


Fig. 8: Normalized confinement radii, (r/R), as a function of Polywell™ radii R m. (a) Four electron energies with 10 kA current in the coils. (b) Four electron energies with 100 kA current in the coils.

$$\frac{r_m}{R} = \alpha\left[\exp\left(-4.64\times 10^{-4}\frac{IR}{K^{\frac{1}{3}}}\right) - 1 + \frac{0.88}{\alpha}\right]$$

$$\alpha = -0.039\ln\left(K\right) + 0.512$$

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Future and current work

Simulation improvements:

- Space charge effects need to be taken into account.
- Modeling central virtual cathode potential.
- Space charge limited flow in the cusps.

For collisionless plasma modeling Poisson-Vlasov equation needs to be solved.

Investigation of the diamagnetic interaction of the plasma with the vacuum fields.

Can a 1D model of the magnetic cusps suffice?